Reconstruction of EEG signal using the thin plate and linear spline radial basis functions

1*st Mahboubeh Molavi-Arabshahi[1], 2nd Mobina Majouni[2]

[1] Department of Mathematics, Iran University of Science and Technology, Iran
[2] Department of Mathematics, Iran University of Science and Technology, Iran

molavi@iust.ac.ir

***Abstract*—To understand the complex structure and function of the brain network, the function of the brain must be considered as an integrated system. And using new tools and methods we can correctly analyze the performance of the system.**
In this study, brain signal (EEG) reconstruction is performed by interpolation based on radial basis functions (RBF), which is suitable for interpolation for scattered data. Due to the importance of correct recording of EEG signal in this paper, we decided to use radial basis function, especially two linear and spline bases to reconstruct this signal. Also, by comparing these two bases and the factors involved in interpolation, such as the number of data, we choose a more efficient basis for signal analysis.

***Keywords*— Electroencephalogram- EEG signal - Radial basis functions (RBF) - Spline basis - Linear basis**

I. Introduction

Understanding the relationship between different areas of the brain is one of the most important topics in neuroscience. There are various methods same as the signal recording method for recognizing the functional relationship of the brain as well as recording the neuronal activity of its different regions. One of the common methods of signal recording for study of how different areas of the brain respond in response to physiological activities is the use of non-invasive electroencephalographic signal recording (EEG). Studies over the past few decades have shown that the use of electroencephalography, along with appropriate signal processing techniques (including EEG signal interpolation), can provide useful information about normal networks of cognitive processes and disease diagnosis.
This is seen as a major challenge in neuroscience, which temporarily seeks to find a very short time following which the brain processes involved in a task are measured. In this regard, over the past few decades, with the aim of identifying different active areas of the brain, extensive studies have been conducted on brain signals.
Sporns et al., 2010 also demonstrated features of brain network organization in the form of quantitative analysis of modeling tools. The results of their studies provide an overview of how networking works and methods related to the anatomy of the human brain on a large scale [1]. Friston et al.,1994 Nodes can when brain networks are examined on a large scale Represent different areas of the brain. According to him; Depending on the type Total data, links represent descriptive, functional or communications
They are effective in the brain. Also, functional connection as much as correlation It is sometimes dependent on brain activity. Because this connection may be on Occurs between pairs of areas of the brain, unrelated to the anatomy of the body [2]. Considering the mentioned cases and the importance of correct perception and analysis of EEG signal in timely diagnosis of diseases that can cause irreversible complications and irreversible damage for individuals. In this study using the interpolation of EEG data by basic functions, we reconstructed the mentioned linear radial and spline basis and then introduced the best reconstruction performed [3].
EEG signal Electroencephalography is the recording of electrical activity in the brain. This technique involves receiving a signal by surface electrodes, improving the signal (usually amplifying and removing noise), printing the signal, and analyzing it. The recorded signal can be either directly or after computer processing should be analyzed by a physician or neurologist .

With the help of electroencephalography, it is possible to determine the amount of that activity in different types of brain activity and identify the areas involved in the brain. As a result, the study and analysis of the signal recorded through electroencephalography in a wide range of diagnostic and research applications, such as:

1- Diagnosing cerebral brain injury and determining its location
2- Investigating epileptic seizures
3- Diagnosis of mental disorders
4- Study of sleep and its disorders
5 - Observation and analysis of brain responses to sensory stimuli and .... [4].

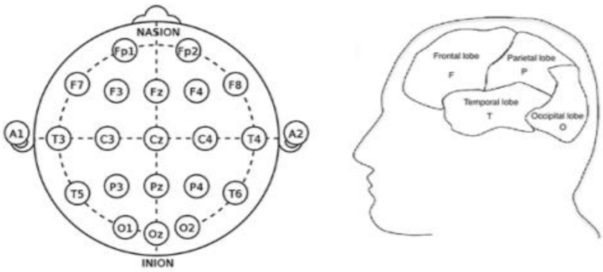

FIG 1. Standard 10-20 system montage. Location and nomenclature of the electrodes: temporal lobe (T), parietal lobe (P), occipital lobe (O), and frontal lobe (F). The odd number corresponds to the left hemisphere and the even number to the right hemisphere

II. METHODOLOGY

A. Interpolation with radial basis functions:

One of the important discussions of applied mathematics is the approximation of functions. For approximation, a function such as f is usually written as a linear combination of basic (finite) functions, i.e.:

$$f \cong \overline{f} = \sum_{i=1}^{n} c_i u_i \qquad (1)$$

In this relation, $\{u_1, u_2, ..., u_n\}$ is the basis of definite functions and n is a natural number, and c_i are real coefficients that we must obtain. There are several ways to obtain these coefficients; Here we use interpolation. In the interpolation method, n distinct nodes are usually considered and by applying the interpolation conditions in n nodes, we form the following relations:

$$\lambda_j = f(x_j) = \overline{f}(x_j) = \sum_{i=1}^{n} c_i u_i(x_j) \qquad 1 \le j \le n \qquad (2)$$

(At least one of the $\lambda_j \neq 0$).

Equation (1) is a linear system of n equations and n unknowns which the matrix form is $AC = \lambda$.
The matrix A is called the internal matrix. For $AC = \lambda$ to be solvable, the necessary condition is that A be non-singular.

Table 1: Some important radial basis functions

Radial basis function name	φ (r)	smoothness
Multiquadric	$\varphi(r) = (r^2 + c^2)^{\frac{1}{2}}$	Smooth
Inverse Multiquadric	$\varphi(r) = (r^2 + c^2)^{\frac{-1}{2}}$	Smooth
Thin-plate-spline	$r \ln(x - x_i)$	Smooth
Linear(spline)	r	Smooth

B. Obtain radial basis functions

In interpolation with radial basis functions, a function is approximated using a linear combination of unique radial basis functions such as $\phi(\|.\|)$. In other words, the interpolation form with radial basis functions is as follows: [5].

$$\boldsymbol{S}(\boldsymbol{x}) = \sum_{\boldsymbol{j}=\boldsymbol{1}}^{\boldsymbol{n}} \boldsymbol{\lambda}_{\boldsymbol{j}}\, \boldsymbol{\phi}(\|\boldsymbol{x} - \boldsymbol{x}_{\boldsymbol{j}}\|) \quad (3)$$

III. NUMERICAL RESULTS

To interpret the EEG signal with radial basis functions, we examined 5,000 discrete EEG data obtained from the first 10 seconds of an adult's electroencephalogram [6]. We also examined radial basis functions with two linear and spline bases [7], [8] to find better results. To select the data required for interpolation from among the 5000 primary data, we first selected 1000 data as a multiple of the initial data and interpolated 1000 data with two linear and spline bases.

Then, again, 2500 data were selected from among the initial 5000 data for interpolation with two linear and spline bases, and we compared both mentioned modes to achieve the best result which can be seen as follows:

In Figure (2), we show the EEG signal indicator in the first 10 seconds, based on 5000 primary data with MATLAB software.

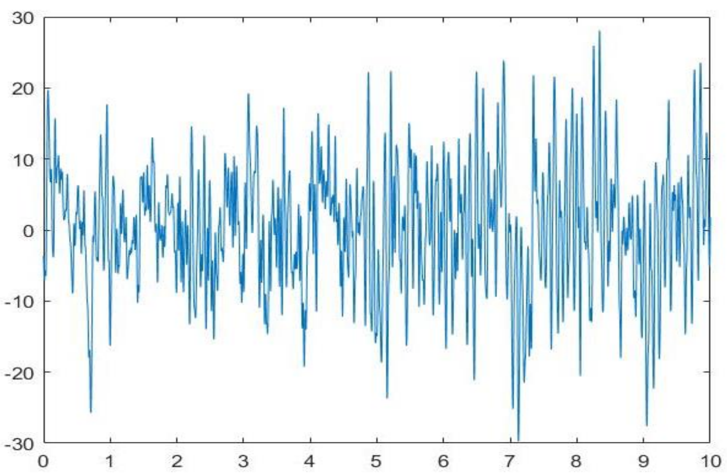

FIG 2. The EEG signal in the first 10 seconds, citing 5000 primary data

Compare the original data with 1000 interpolated data with two linear bases and a spline.

In Figure 3, by selecting 1000 data from 5000 primary data, we plotted the interpolation using two bases of linear spline and thin plate, and also plotted the initial data diagram. As it turns out, the boundary between the main chart and the interpolated chart is small.

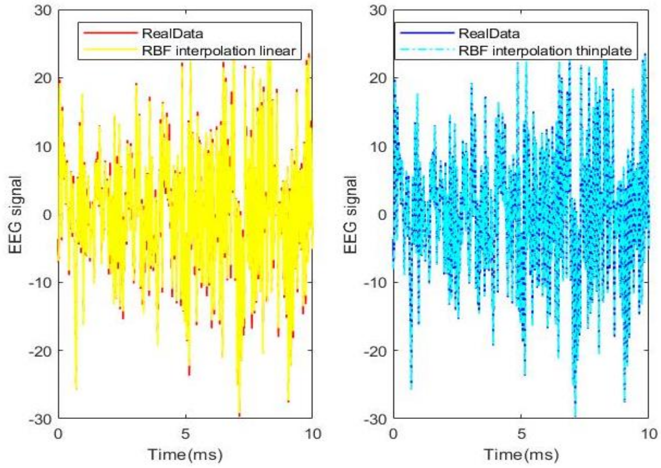

FIG 3. EEG signal diagram with two bases of linear spline and thin plate

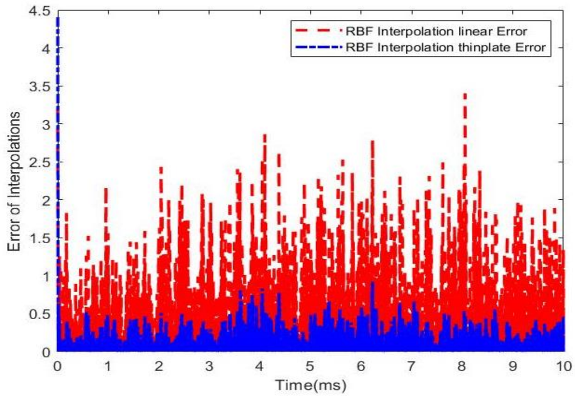

FIG 4. Two interpolation errors with 1000 data and two linear and thin plate are displayed

Interpolation error with the spline basis is less than the linear basis.

Compare the original data with 2500 interpolated data with two linear and spline bases.
In Figure 5, by selecting 2500 data from 5000 primary data, we plot the interpolation using two bases, linear spline and thin plate, and also plot the initial data. As can be seen, the difference between the original data and the internalized data is negligible and the accuracy is much higher than in Figure 3.

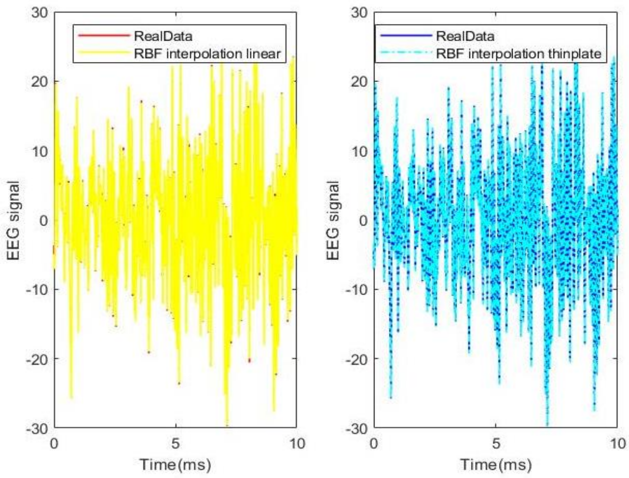

FIG 5. Two interpolation errors with 2500 data and two linear and thin plate are displayed

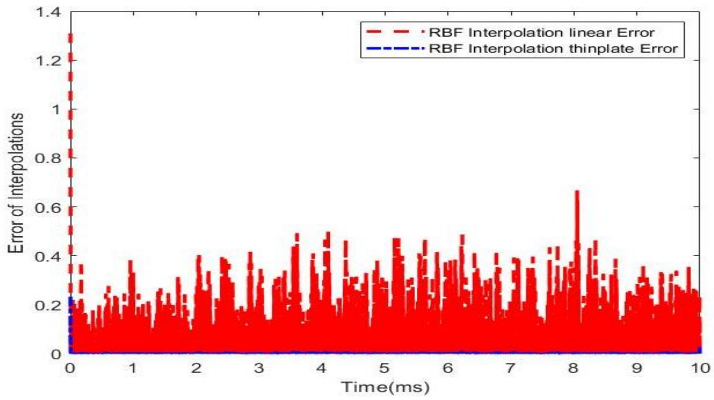

FIG 6. Two interpolation errors with 2500 data and two linear and thin plate are displayed

Interpolation error with the spline basis is less than the linear basis.

Figure 6 shows two interpolation errors performed with 2500 data and two linear and spline pins.
As you can see in the figure 6, the interpolation error with the thin plate basis is less than the linear basis, and because the number of data has increased compared to the previous case, the curve has become more accurate so that the red line related to the spline interpolation has a narrow border that is seen at the bottom of image.

IV. Conclusion

Table 2: Error comparison

Error / Number of data	Max error with Linear basis	Max error with Spline basis
1000	3.4028	4.40464
2500	1.30868	0.233191

As shown in Table 2, as the number of data increases, our interpolation becomes more accurate and the overall error with each base of the radial basis functions decreases.
Also, when we used 1000 data for interpolation, the error with the linear basis was reduced, but when the interpolation was done with 2500 data then the interpolation was more accurate, the spline basis calculated a smaller and very good error.
Therefore, with the help of interpolation and reconstruction of basic radial functions, we were able to reconstruct the EEG signal with a very good approximation and low error [9].

References

[1] M. Rubinov and O. Sporns, Complex network measures of brain connectivity: Uses and interpretations, Neuroimage, vol. 52, no. 3, pp. 1059–1069, 2010

[2] K. J. Friston, Functional and effective connectivity in neuroimaging: A synthesis, Hum. Brain Mapp. vol. 2, no. 1–2, pp. 56–78, 1994

[3] J.C. Carr, R.K. Beatson, J.B. Cherrie, T.J. Mitchell, W.R. Fright, B.C. McCallum and T.R. Evans, "Reconstruction and representation of 3D objects with radial basis functions", Computer Graphics (SIGGRAPH 2001 proceedings), pp. 67- 76, August 2001.

[4] M. Sharanreddy and P. K. Kulkarni "Can EEG test help in identifying brain tumor?", Int Sch Sci Res Innov. 2013;7(11):703-8.

[5] B.J. Ch Baxter, "The Interpolation Theory of Radial Basis Functions, PhD thesis, Trinity College, Cambridge University, U.K., 1992

[6] www.physionet.org.

[7] C-H Bi, L. Geng and X-Z. Zhang, "Cubic spline interpolation-based time-domain equivalent source method for modeling transient acoustic radiation. Elsevier (Journal of Sound and Vibration), 2013 Oct; 332(22): 5939–5952.

[8] J. Duchon, "Splines minimizing rotation-invariant semi-norms in Sobolev space", Constructive Theory of Functions of Several Variables, Springer Lecture Notes in Math, 21 (1977), pp. 85-100.

[9] G.B. Wright, "Radial Basis Function Interpolation: Numerical and Analytical Developments", University of Colorado, Thesis, 2003

بازسازی سیگنال مغز با استفاده از توابع پایه ای شعاعی اسپلاین هموار و اسپلاین خطی

محبوبه مولوی عربشاهی*[1]، مبینا معجونی[2]

[2,1]دانشکده ریاضی، دانشگاه علم و صنعت ایران، تهران، ایران

[2,1]دانشکده ریاضی، دانشگاه علم و صنعت ایران، تهران، ایران

molavi@iust.ac.ir

چکیده برای شناخت ساختار و عملکرد پیچیده شبکه مغز، باید عملکرد مغز، به عنوان یک سیستم یکپارچه در نظر گرفته شود الگوهای به دست آمده از عملکرد نواحی مختلف مغز و تکنیک های پردازش آن، مجموعه کاملی از اطلاعات مربوط به ارتباط موجود میان نواحی مغز را گردآوردی و با بهره گیری از ابزارها و روش های جدید، به تحلیل صحیح عملکرد سیستم آن می پردازد.

در این پژوهش بازسازی سیگنال مغز (EEG) با درون یابی بر اساس توابع پایه شعاعی (RBF) که برای درون یابی داده های پراکنده مناسب است انجام می شود با توجه به اهمیت ثبت درست سیگنال EEG در این مقاله برآن شدیم تا با استفاده از توابع پایه ای شعاعی به ویژه دو پایه خطی و اسپلاین به بازسازی این سیگنال بپردازیم. همچنین با مقایسه این دو پایه و عوامل دخیل در درونیابی مانند تعداد داده،پایه کارآمد تر برای بررسی و تحلیل سیگنال را انتخاب می کنیم.

کلید واژگان: الکتروانسفالوگرام – سیگنال EEG – توابع پایه ای شعاعی (RBF) – پایه اسپلاین – پایه خطی