Reconstruction of ECG signal using multiquadric and gaussian radial basis functions

1*st Mahboubeh Molavi-Arabshahi[1], 2nd Zahra Khorshidi[2]

[1] Department of Mathematics, Iran University of Science and Technology, Iran
[2] Department of Mathematics, Iran University of Science and Technology, Iran
molavi@iust.ac.ir

***Abstract*—Since the human heart is one of the most important organs of the body, many researchers have always tried to use different mathematical methods to reconstruct heart signals. Electrocardiogram signal (ECG) is often used to diagnose heart disease, and the ECG signal is usually a combination of a signal and an interference that may complicate computer interpretation. These interferences, which are mostly caused by body and eye movements, cause error in signal recording and analysis and can greatly limit the use of the ECG signal. For this reason, in this study, a new method has been used to reconstruct the ECG signal. Common methods used in the past have included wavelet, Fourier, spline interpolation, etc., each of them has its own advantages and disadvantages. The method used in this study is to reconstruct and perform interpolation operations using multiquadric and Gaussian radial basis functions, which, using the applied data, give us a good approximation of its wave form.**

***Keywords*— Electrocardiogram, Reconstruction of signal, Interpolation, multiquadric Basic Function, Gaussian Basic Function**

I. Introduction

Maintaining human health is the most important and valuable feature that human beings are willing to do even hard work to achieve [1]. One of these tasks is to build different devices and improve them over time.

Among the causes of death between the countries of the world, heart diseases are the most common [2]. The heart is one of the most important organs that any abnormality in its behavioral rhythm overshadows human health.

The ECG signal works by receiving and amplifying small electrical changes made to the skin during each cardiac muscle depolarization procedure. The recorded potential difference between two electrodes placed on the surface of the skin is known as a surface electrocardiogram. Depolarization, successive atrial and ventricular re-polarizations that occur during each cardiac cycle, create peaks and valleys in a single normal ECG cycle; these peaks and valleys are named with the letters P, Q, R, S, T (Bruce,2001).

Analysis of cardiac signal behavior as a well-behaved function, using data analysis, has always been of interest to many researchers around the world. In addition to the heart, biological signal responses are seen in many natural organs and systems, such as the brain and cell membranes, which generally behave in a regular and repetitive manner [3]. A heart rate that deviates slightly from its path is a possible cause of heart disease. Therefore, researchers are trying to find ways to analyze electrocardiograms, which are the same signals produced by the heart's electrical potential. They also try to develop mathematical models that reconstruct these basic properties.

Some research on heart disease-Time series modeling is dynamic research that has attracted the attention of the research community for more than a few decades.

Kumari et al. [4,5] analyzed the cardiovascular disease dataset using data mining, classification, decision tree, and support vector machines.

Guru et al. [6] Used neural networks to predict heart disease, hypertension, and blood sugar.

In this study, we first reconstruct the electrocardiogram signal using the multiquadric and Gaussian basis function, and then perform the interpolation operation using the obtained function. Finally, we consider the accuracy of the method by analyzing the results.

The required ECG data for this study were obtained from www.physionet.org [7].

ECG signal-One of the oldest and simplest non-invasive methods of diagnosing cardiovascular disease is electrocardiography. An electrocardiograph, called an ECG or EKG, has been one of the most important tools for diagnosing heart disease since the 19th century [8].
The electrocardiograph continuously records this diagram on a dedicated striped paper tape. The information recorded on the electrocardiogram shows the electrical waves of the heart. These waves indicate different stages of heart stimulation.
The curve that is shown is called an electrocardiogram. Doctors can use this curve to find out how the heart works. Each curve contains three waves. The p-wave represents the ventricular electrical activity just before the atria, the QRS complex represents the ventricular electrical activity just before the ventricles, and the T wave represents the ventricular relaxation.

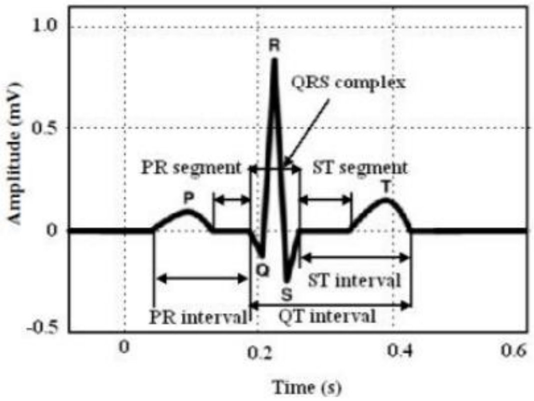

FIG.1. A sample of a normal ECG waveform

II. METHODOLOGY

In many areas, we are encounter with the issue that a number of scattered data are provided to us and we are asked to find a relationship and, in fact, a criterion between these data, which analysts find them is very important.
In this research for finding this relationship, we applied basic radial functions.
Radial function- Suppose x, y is belong to the arbitrary vector space X and $\|.\|$ is a norm on X, the real function of the value of F is called radial on X, if $\|x\| = \|y\|$ then:

$$F(x) = F(y)$$

Consequently, there is a function such as $f : R^+ \to R$ so that:

$$F(x) = f(\|x\|)$$

We consider vector space as R^s and norm as Euclidean norm.
Basic radial functions-A set of functions $B = \{F_1, F_2, ..., F_n\}$ is called a radial basis function whenever B is a basis and each of the F_i is a radial function.

Table 1: Some important radial base functions		
Radial base function name	φ (r)	Conditions
Gaussian	$Exp(-c^2 \times r^2)$	$c \succ 0$
Multiquadric	$\varphi(r) = (r^2 + c^2)^{\frac{1}{2}}$	$c \succ 0$
Inverse Multiquadric	$\varphi(r) = (r^2 + c^2)^{\frac{-1}{2}}$	$c \succ 0$
Thin-plate-spline	$r \ln(x - x_i)$	------
Linear-spline	r	------

Interpolation with radial basis functions-One of the important discussions of applied mathematics is the approximation of functions. For approximation, a function such as f is usually written as a linear combination of basic (finite) functions, i.e.:

$$f \cong \overline{f} = \sum_{i=1}^{n} c_i u_i \tag{1}$$

In this relation, $\{u_1, u_2, ..., u_n\}$ is the basis of definite functions and n is a natural number, and c_i are real coefficients that we must obtain. To obtain these coefficients, we use the interpolation method. Equation (1) is a linear system of n equations and n unknowns whose matrix form is $AC = \lambda$. So that:

$$A = \begin{pmatrix} u_1(x_1) & \dots & u_n(x_1) \\ \vdots & \ddots & \vdots \\ u_1(x_n) & \cdots & u_n(x_n) \end{pmatrix}, \quad C = \begin{pmatrix} c_1 \\ c_2 \\ . \\ . \\ c_n \end{pmatrix}, \quad \lambda = \begin{pmatrix} \lambda_1 \\ \lambda_2 \\ . \\ . \\ \lambda_n \end{pmatrix}$$

The matrix A is called the interpolation matrix. For $AC = \lambda$ to be solvable, the necessary condition is that A be non-singular.

Obtain radial basis functions-In interpolation with radial basis functions, a function is approximated using a linear combination of unique radial basis functions such as $\phi(\|.\|)$. In other words, the interpolation form with radial basis functions is as follows [9]:

$$S(x) = \sum_{j=1}^{n} \lambda_j \phi(\|x - x_j\|) \tag{2}$$

III. NUMERICAL RESULTS

We reconstructed and internalized the desired ECG signal using multiquadric radial basis function with $c = 0.1$ and Gaussian radial basis function with $c = 1$ in MATLAB software. The results are shown as follows:
(The total number of data is 350 that we used to internalize 70 data once and 175 data again).

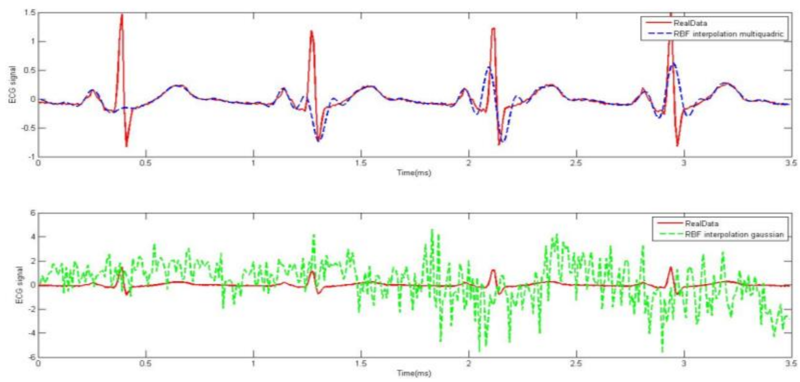

FIG.2. Interpolation with 70 selected data

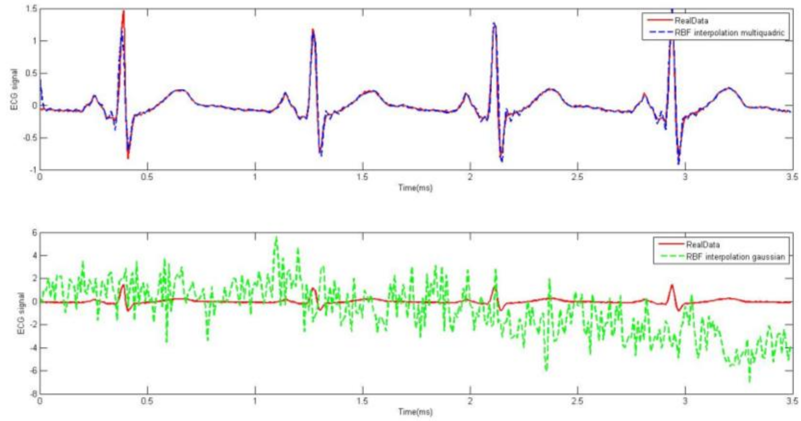

FIG.3. Interpolation with 175 selected data

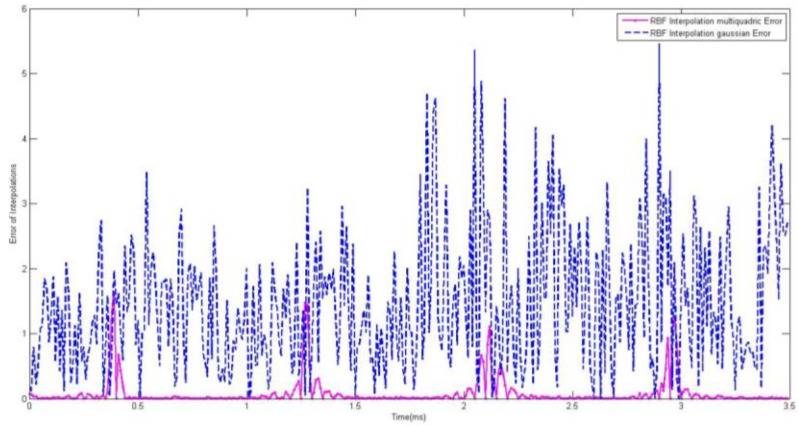

FIG.4. Comparison of errors in two radial bases with 70 sample data

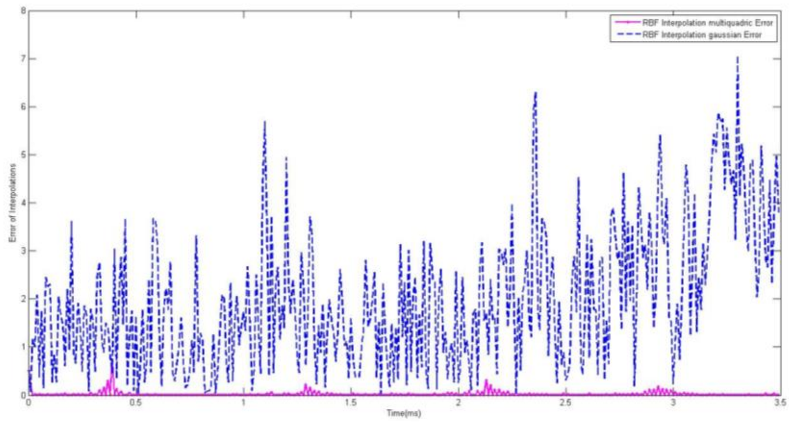

FIG.5. Comparison of errors in two radial bases with 175 sample data

Table 2: Error analysis

Number of selected data	Maximum interpolation error with Multiquadric radial base function ($c = 0.1$)	Maximum interpolation error with Gaussian radial basis function ($c = 1$)
70	1.63381	5.46303
175	0.708178	7.06498

IV. Analysis and Conclusion

According to Figure 2 (interpolation with 70 selected data), we find that the signal reconstructed by the multiquadric radial base with $c = 0.1$ is more in line with the original signal than the signal reconstructed by the Gaussian radial base with $c = 1$, which in Figure 4 of this difference is visible.

Figure 3 (interpolation with 175 selected data) also shows that signal reconstruction with a multiquadric radial base with $c = 0.1$ is more accurate than Gaussian radial base, which can be seen in Figure 5.

Therefore, the multiquadric radial base function with $c = 0.1$ is more accurate and more suitable for reconstructing the existing ECG signal than the Gaussian radial base with $c = 1$.

Also, according to Table 2, we find that if we increase the number of points used in interpolation (for example, in this study from 70 data to 175 data), the accuracy of interpolation by the multiquadric radial base with $c = 0.1$, increases.

References

[1] Baig MM, Gholamhosseini H. Smart Health Monitoring Systems: An Overview of Design and Modeling. Springer (Journal of Medical Systems), 2013 Jan; 37(2): 1-14.

[2] Michimi, A. Ellis-Griffith, G. Nagy, C. Peterson, T. (2013), “Coronary heart disease prevalence and occupational structure in U.S. metropolitan areas: A multilevel analysys,” Health & place, 21, pp 192-204.

[3] Wolpaw, J. R. Birbaumer, N. McFarland, D. J. Pfurtscheller,G. Vaughan,T.M. (2002) , “Brain-computer interfaces for communication and control,” Clinical Neurophysiology, 113, pp 767-791.

[4] P.C. Chang, Y.W. Wang, Fuzzy Delphi and back-propagation model for sales forecasting in PCB industry, Expert Systems with Applications 30 (4) (2006)715–726.

[5] R.K. Lai, C.-Y. Fan, W.-H. Huang, P.-C. Chang, Evolving and clustering fuzzy decision tree for financial time series data forecasting, Expert Systems with Applications 36 (2–2), 2009, 3761–3773.

[6] János Abonyi, Ferenc Szeifert, Supervised fuzzy clustering for the identification of fuzzy classifiers, Pattern Recognition Letters 24 (14), 2003, 2195–2207.

[7] www.physionet.org.

[8] AlGhatrif, M. Lindsay, J. (2014) “Abrief review: history to understand fundaments of electrocardiography,” journal of community Hospital internal perspectives, https://www.researchgate.net/publication/251570400_A_brief_review_history_to_u nderstand_fundamentals_of_electrocardiography.

[9] M. Buhmann. Radial Basis Functions, Theory and Implementations. Cambridge University Press, 2003.

بازسازی سیگنال قلبی با استفاده از توابع پایه ای شعاعی چند مربعی و گاوسین

محبوبه مولوی عربشاهی[۱*]، زهرا خورشیدی[۲]

[۱] دانشکده ریاضی، دانشگاه علم و صنعت ایران، تهران، ایران

[۲] دانشکده ریاضی، دانشگاه علم و صنعت ایران، تهران، ایران

molavi@iust.ac.ir

چکیده: از آن جایی که قلب انسان یکی از مهم ترین ارکان بدن به شمار می آید، همواره محققان بسیاری در تلاش بوده اند روش های ریاضی مختلفی را جهت باز سازی سیگنال های قلبی به انجام برسانند. سیگنال الکتروکاردیوگرام (ECG) اکثراً برای تشخیص بیماریهای قلبی استفاده میشود و معمولا سیگنال ECG ترکیبی است از یک سیگنال و یک تداخل که ممکن است تفسیر کامپیوتری را پیچیده سازد، این تداخل ها که اکثراً ناشی از حرکات بدن و چشم میباشد باعث ایحاد خطا در ثبت سیگنال و همچنین تحلیل آن میشود و میتواند کاربرد سیگنال ECGرا تا حد زیادی محدود کند.به همین دلیل در این پژوهش از روشی نوین جهت بازسازی سیگنال ECG استفاده شده است. روش های متداول صورت گرفته در گذشته شامل ویولت، فوریه، درون یابی اسپلاین و ... بوده اند که هر یک مزایا و معایب مخصوص به خود را داشته اند. روش صورت گرفته در این مطالعه بازسازی و انجام عمل درونیابی با استفاده از توابع پایه ای شعاعی چند مربعی (Multiquadrics) و گاوسین (Gaussian) می باشد که با استفاده از داده های اعمال شده، تقریب مناسبی از شکل موج آن آنها را به ما می دهد.

کلید واژگان: الکتروکاردیوگرام، بازسازی سیگنال، درونیابی، تابع پایه ای شعاعی چند مربعی (Multiquadrics)، تابع پایه ای گاوسین (Gaussian)