Reconstruction of ECG signal using multiquadric and gaussian radial basis functions

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***Abstract*—Since the human heart is one of the most important organs of the body, many researchers have always tried to use different mathematical methods to reconstruct heart signals. Electrocardiogram signal (ECG) is often used to diagnose heart disease, and the ECG signal is usually a combination of a signal and an interference that may complicate computer interpretation. These interferences, which are mostly caused by body and eye movements, cause error in signal recording and analysis and can greatly limit the use of the ECG signal. For this reason, in this study, a new method has been used to reconstruct the ECG signal. Common methods used in the past have included wavelet, Fourier, spline interpolation, etc., each of them has its own advantages and disadvantages. The method used in this study is to reconstruct and perform interpolation operations using multiquadric and Gaussian radial basis functions, which, using the applied data, give us a good approximation of its wave form.**

Keywords— Electrocardiogram, Reconstruction of signal, Interpolation, multiquadric Basic Function, Gaussian Basic Function

# Introduction

Maintaining human health is the most important and valuable feature that human beings are willing to do even hard work to achieve [1]. One of these tasks is to build different devices and improve them over time.

Among the causes of death between the countries of the world, heart diseases are the most common [2]. The heart is one of the most important organs that any abnormality in its behavioral rhythm overshadows human health.

The ECG signal works by receiving and amplifying small electrical changes made to the skin during each cardiac muscle depolarization procedure. The recorded potential difference between two electrodes placed on the surface of the skin is known as a surface electrocardiogram. Depolarization, successive atrial and ventricular re-polarizations that occur during each cardiac cycle, create peaks and valleys in a single normal ECG cycle; these peaks and valleys are named with the letters P, Q, R, S, T (Bruce,2001).

Analysis of cardiac signal behavior as a well-behaved function, using data analysis, has always been of interest to many researchers around the world. In addition to the heart, biological signal responses are seen in many natural organs and systems, such as the brain and cell membranes, which generally behave in a regular and repetitive manner [3]. A heart rate that deviates slightly from its path is a possible cause of heart disease. Therefore, researchers are trying to find ways to analyze electrocardiograms, which are the same signals produced by the heart's electrical potential. They also try to develop mathematical models that reconstruct these basic properties.

**Some research on heart disease**-Time series modeling is dynamic research that has attracted the attention of the research community for more than a few decades.

Kumari et al. [4,5] analyzed the cardiovascular disease dataset using data mining, classification, decision tree, and support vector machines.

Guru et al. [6] Used neural networks to predict heart disease, hypertension, and blood sugar.

In this study, we first reconstruct the electrocardiogram signal using the multiquadric and Gaussian basis function, and then perform the interpolation operation using the obtained function. Finally, we consider the accuracy of the method by analyzing the results.

The required ECG data for this study were obtained from [www.physionet.org](http://www.physionet.org) [7].

**ECG signal**-One of the oldest and simplest non-invasive methods of diagnosing cardiovascular disease is electrocardiography. An electrocardiograph, called an ECG or EKG, has been one of the most important tools for diagnosing heart disease since the 19th century [8].

The electrocardiograph continuously records this diagram on a dedicated striped paper tape. The information recorded on the electrocardiogram shows the electrical waves of the heart. These waves indicate different stages of heart stimulation.

The curve that is shown is called an electrocardiogram. Doctors can use this curve to find out how the heart works. Each curve contains three waves. The p-wave represents the ventricular electrical activity just before the atria, the QRS complex represents the ventricular electrical activity just before the ventricles, and the T wave represents the ventricular relaxation.

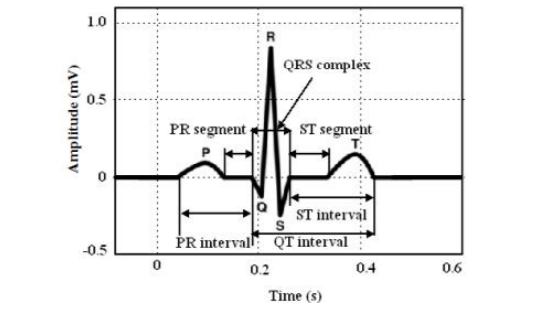


FIG.1. A sample of a normal ECG waveform

# Methodology

In many areas, we are encounter with the issue that a number of scattered data are provided to us and we are asked to find a relationship and, in fact, a criterion between these data, which analysts find them is very important.

In this research for finding this relationship, we applied basic radial functions.

**Radial function-** Suppose is belong to the arbitrary vector space  and  is a norm on , the real function of the value of  is called radial on , if  then:



Consequently, there is a function such as  so that:



We consider vector space as  and norm as Euclidean norm.

**Basic radial functions-**A set of functions  is called a radial basis function whenever  is a basis and each of the  is a radial function.

|  |  |  |
| --- | --- | --- |
| **Table 1: Some important radial base functions** | | |
| Radial base function name | (r) | Conditions |
| Gaussian |  |  |
| Multiquadric |  |  |
| Inverse Multiquadric |  |  |
| Thin-plate-spline |  | ------ |
| Linear-spline |  | ------ |

**Interpolation with radial basis functions**-One of the important discussions of applied mathematics is the approximation of functions. For approximation, a function such as f is usually written as a linear combination of basic (finite) functions, i.e.:

|  |  |
| --- | --- |
| (1) |  |

In this relation,  is the basis of definite functions and  is a natural number, and  are real coefficients that we must obtain. To obtain these coefficients, we use the interpolation method. Equation (1) is a linear system of  equations and  unknowns whose matrix form is . So that:



The matrix  is called the interpolation matrix. For  to be solvable, the necessary condition is that  be non-singular.

**Obtain radial basis functions-**In interpolation with radial basis functions, a function is approximated using a linear combination of unique radial basis functions such as . In other words, the interpolation form with radial basis functions is as follows [9]:

|  |  |
| --- | --- |
| (2) |  |

# Numerical results

We reconstructed and internalized the desired ECG signal using multiquadric radial basis function with  and Gaussian radial basis function with  in MATLAB software. The results are shown as follows:

(The total number of data is 350 that we used to internalize 70 data once and 175 data again).

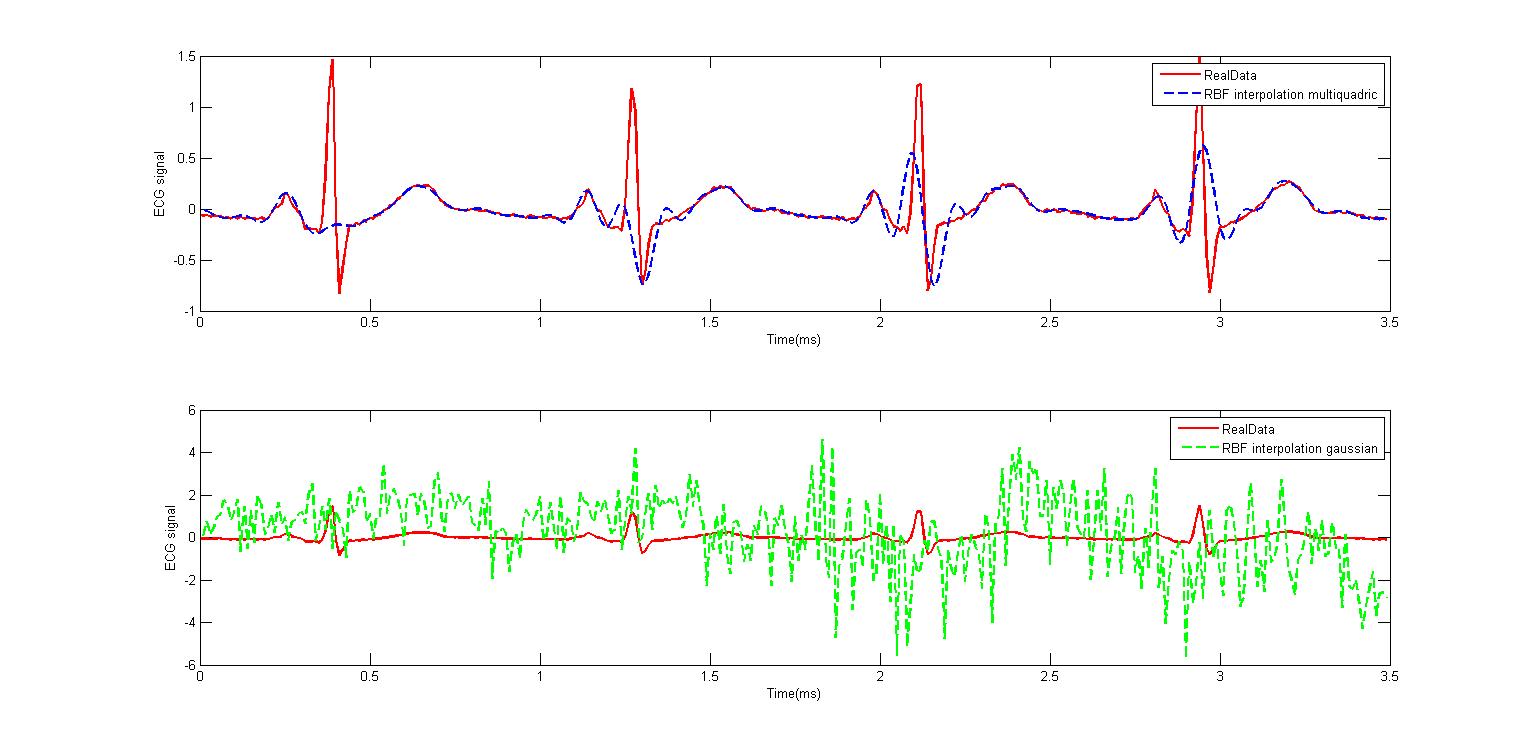


FIG.2. Interpolation with 70 selected data

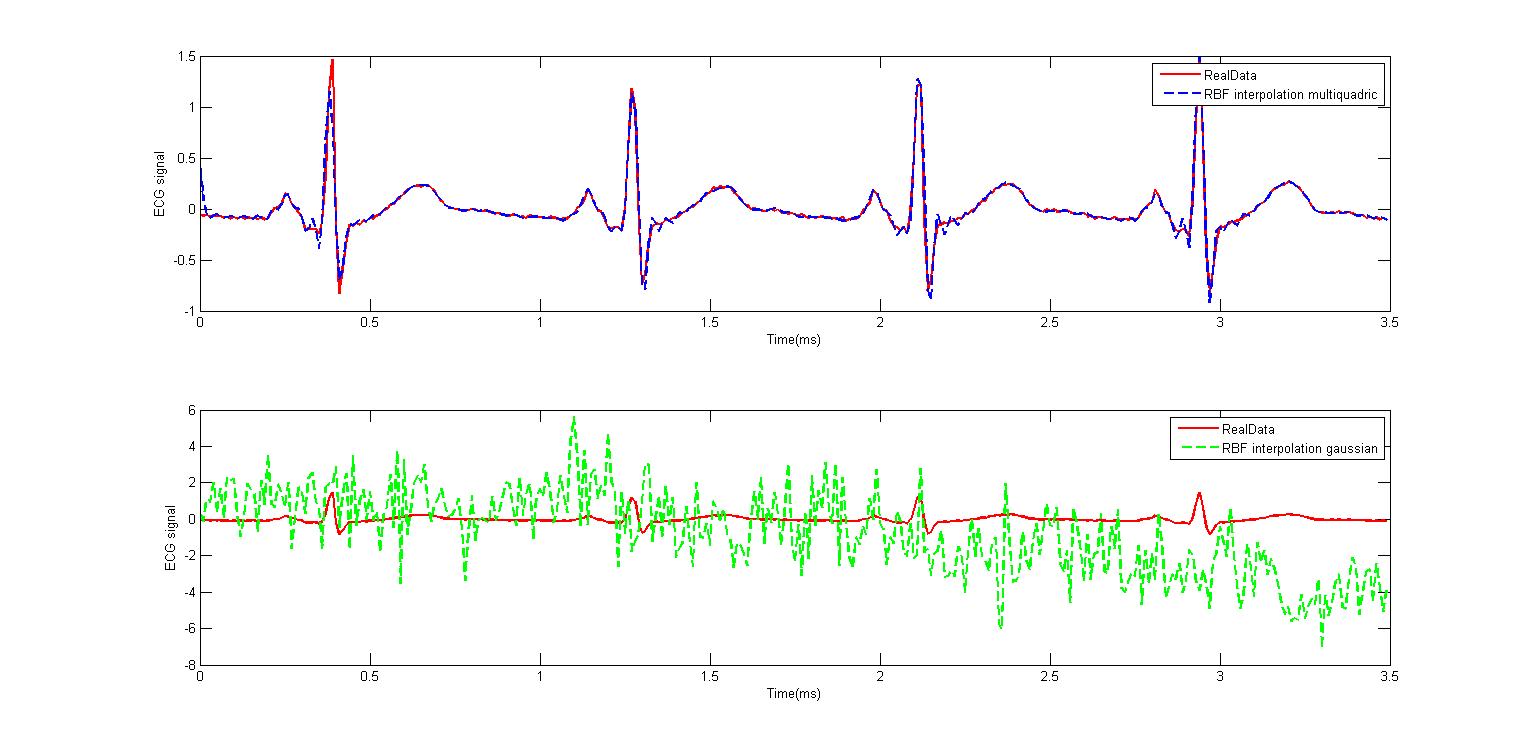


FIG.3. Interpolation with 175 selected data

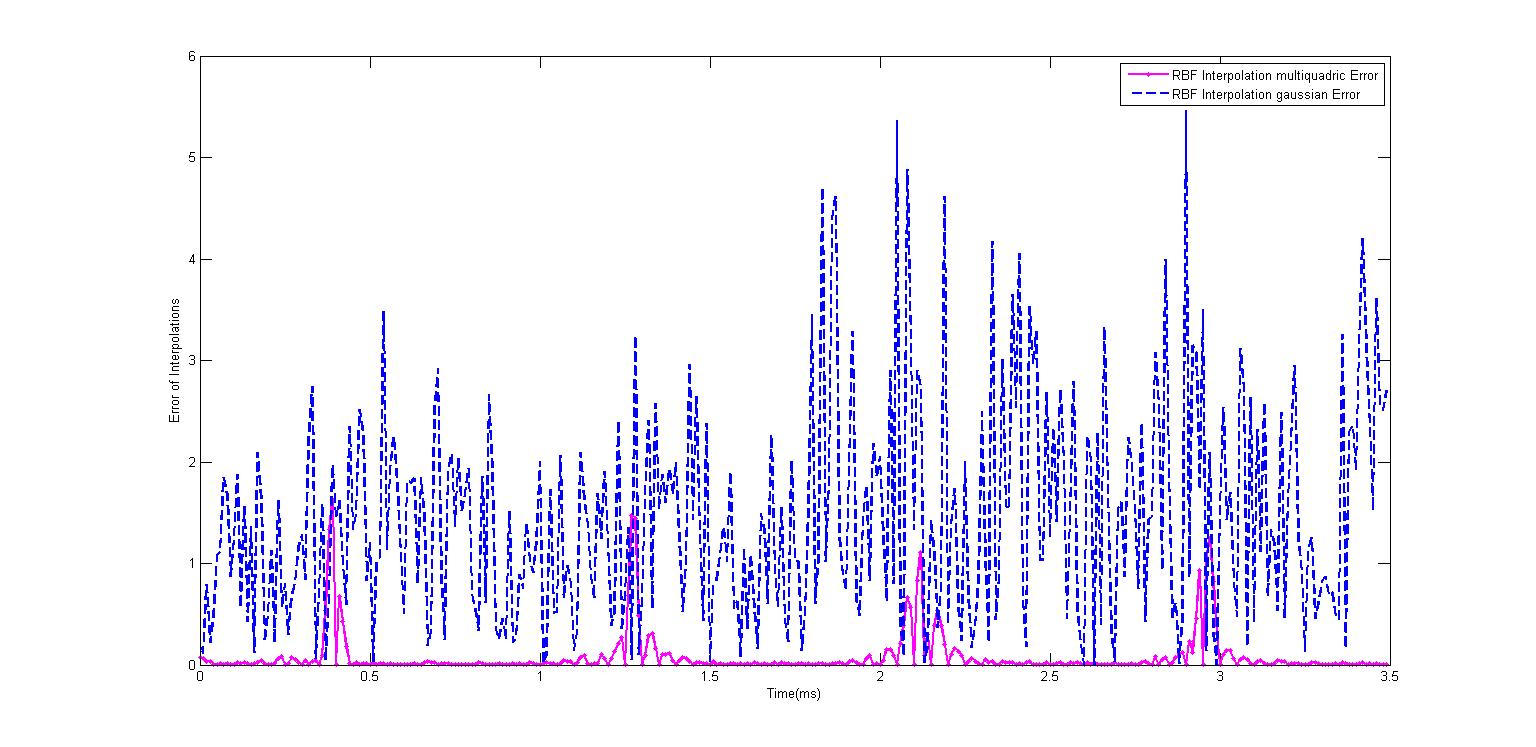


FIG.4. Comparison of errors in two radial bases with 70 sample data

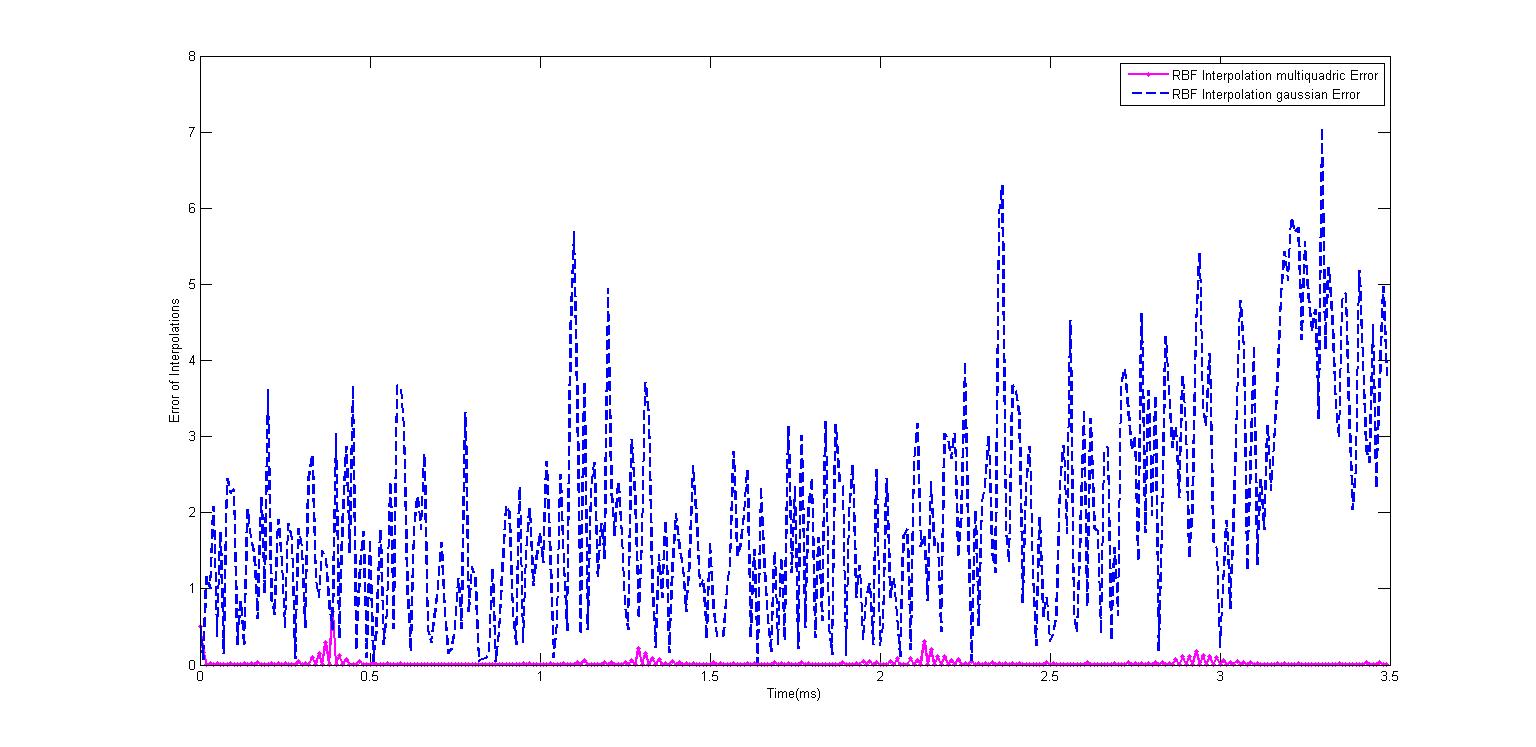


FIG.5. Comparison of errors in two radial bases with 175 sample data

|  |  |  |
| --- | --- | --- |
| **Table 2: Error analysis** | | |
| Number of selected data | Maximum interpolation error with Multiquadric radial base function () | Maximum interpolation error with Gaussian radial basis function () |
| 70 | 1.63381 | 5.46303 |
| 175 | 0.708178 | 7.06498 |

# Analysis and Conclusion

According to Figure 2 (interpolation with 70 selected data), we find that the signal reconstructed by the multiquadric radial base with  is more in line with the original signal than the signal reconstructed by the Gaussian radial base with , which in Figure 4 of this difference is visible.

Figure 3 (interpolation with 175 selected data) also shows that signal reconstruction with a multiquadric radial base with  is more accurate than Gaussian radial base, which can be seen in Figure 5.

Therefore, the multiquadric radial base function with  is more accurate and more suitable for reconstructing the existing ECG signal than the Gaussian radial base with .

Also, according to Table 2, we find that if we increase the number of points used in interpolation (for example, in this study from 70 data to 175 data), the accuracy of interpolation by the multiquadric radial base with , increases.

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بازسازی سیگنال قلبی با استفاده از توابع پایه ای شعاعی چند مربعی و گاوسین

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**چکیده**: از آن جايي كه قلب انسان يكي از مهم ترين اركان بدن به شمار مي آيد، همواره محققان بسياري در تلاش بوده اند روش هاي رياضي مختلفي را جهت باز سازي سيگنال هاي قلبي به انجام برسانند. سیگنال الکتروکارديوگرام (ECG) اکثراً برای تشخیص بیماریهای قلبی استفاده میشود و معمولا سیگنال ECG ترکیبی است از يک سیگنال و يک تداخل که ممکن است تفسیر کامپیوتری را پیچیده سازد، اين تداخل ها که اکثراً ناشی از حرکات بدن و چشم میباشد باعث ايحاد خطا در ثبت سیگنال و همچنین تحلیل آن میشود و میتواند کاربرد سیگنال ECG را تا حد زيادی محدود کند.به همين دليل در اين پژوهش از روشي نوين جهت بازسازی سيگنال ECG استفاده شده است. روش هاي متداول صورت گرفته در گذشته شامل ويولت، فوريه، درون يابي اسپلاين و ... بوده اند كه هر يك مزايا و معايب مخصوص به خود را داشته اند. روش صورت گرفته در اين مطالعه بازسازی و انجام عمل درونیابی با استفاده از توابع پایه ای شعاعی چند مربعی (Multiquadrics) و گاوسین (Gaussian) مي باشد كه با استفاده از داده هاي اعمال شده، تقريب مناسبي از شكل موج آن آنها را به ما می دهد.

**کلید واژگان:** الکتروکارديوگرام، بازسازی سیگنال، درونیابی، تابع پایه ای شعاعی چند مربعی (Multiquadrics)، تابع پایه ای گاوسین (Gaussian)