Numerical solution for prey and predator

 problem by OHAM and MOHAM

Roya Montazeri

Department of Mathematics, Payame Noor University, P.O. Box 19395-3697, Tehran, Iran

montazer.ro@gmail.com

*Abstract*—**In this research, the predator-prey problem being presented and a new approach based on an alteration of usage of optimal homotopy asymptotic method (OHAM), called multistage optimal homotopy asymptotic method (MOHAM) is utilized to derive an approximate solution of the system of nonlinear volterra differential equations governing on the problem. . The results will be compared to reveal the advantages of the later approach.**

Keywords— Optimal homotopy asymptotic method (OHAM), Multistage optimal homotopy asymptotic method MOHAM), Series solutions

***1. Introduction***

 All models of biological systems are essentially based on systems of nonlinear ordinary differential equations (ODEs).Both mathematical modeling and simulation are very important in recent studies of biological mathematics. In this paper, we study the mathematical model of the pray and predator problem in with some foxes and rabbits are considered living together. Foxes eat the rabbits and rabbits eat clover. suppose that there are enough clovers and the rabbits have enough food to eat. when there are a lot of rabbits, the foxes also grow and their population increase. when the number of foxes of increase ad they eat a lot of rabbits they enter into a short period of food and their number decrease. As the number of the foxes decrease, the rabbits will be safe and their population increase. when the number of rabbits increase the number of foxes would increase and by passing the time, we can see an infinite repeat ability of increase and decrease in the population of these two kinds of animals.

The governing equations to the problem would be as follows[1,2]

where and are represent the populations of rabbits an the foxes at the time t.

The equivalent canonical for Eqs. (1) is as follow

**2**. **OHAM and MOHAM**

 The OHAM approach is usually applied to solve boundary value problems, say ; [1,

 Where is a linear operator, denotes independent variable, is known function, is an unknown function, is a nonlinear operator and is a boundary operator. According OHAM we constructer a homotopy for(2), as the following

 Where is an embedding parameter, is a non-zero auxiliary function and is an unknown function. For and we have and respectively. Thus, as increases from to , the solution varies from to the solutions where is an initial guess, for the solution, that satisfies the linear operator which is obtained from (3), for ;

The auxiliary function is chosen in the form

Whereare constants that will be determined shortly.

To find the solution, can expand as a power series about

 Now, substitution in (3) from (6) and equating the coefficients of the terms with identical powers of , lead to governing equations of, forwhich starts from (4) and followed by;

Where is the coefficient ofin the expansion of

about the embedding parameter

 Whereis given by Eq.(6)

Study of rate of convergence of the series (6) depends upon the auxiliary constants

If the series (6) converges for, one has

Then the th order approximation can be denoted as follows

Substitution of (11) into Eq.(1), results in the following expression for the residual

 If, thenwill be the exact solution and this, in general, does not happen especially in nonlinear problems. In order to find the optimal values of we apply least squares minimization approach

Where

And and are two values, depending on the given problem. Knowing from (13), the approximate solution of order is well determined easily.

If the interval of changes of the time variable is long, then OHAM fails to reach accurate solutions.

 MOHAM overcomes this shortcoming by partitioning the time interval, into subintervals where and OHAM will be applied over each subintervals. The solution at the last point, in each subinterval, denotes an initial approximation to the solution, over the next interval. The process will continue until we achieve the pre-assigned time, 

Implementation of MOHAM is almost the same as OHAM, with some minor changes:

Equations (5), (11), (12), (13), and (14), change to, (17), (18), (19), (21), and (20), respectively. Also, initial approximation inwill be considered as

In addition, deformation equation in each subinterval for will change to the following [1]

Moreover, auxiliary function will be generalized as follows,

The length of the subinterval is apparentlyand the number of subintervals is. Now, we consider derivatives of (20) with respect to to zero. In fact we define in each subinterval . Therefore, the convergence control parameters can be determined from the solution of the following system of equations

Approximate analytic solutions, on each subinterval, are as follows

***3. Numerical results and discussion***

 Here, we are going to demonstrate the efficiency of MOHAM by illustrative prey an predator problem. The results of OHA and MOHA methods will be compared and computations will be performed by Matlab Package.[3,4]

 *For Numerical study the following values are used.*

 *Case1.*

*Case2.*

*Case2.*

 *Case4.*

Following aforementioned OHAM results in

*Case1.*

*17.24383t+40,*

*Case2.*

*Case3.*

*Case4.*

Following aforementioned MOHAM results in

*Case 1. B*y considering , for up to First-order MOHAM approximate solutions and two-order OHAM approximate solutions can be compared, and plots is presented in Figure1.

*Case 2. B*y considering , for up to first-order MOHAM approximate solutions and two-order OHAM approximate solutions can be compared, and plots is presented in Figure2.

*Case 3. B*y considering , for up to first-order MOHAM approximate solutions and two-order OHAM approximate solutions can be compared, and plots is presented in Figure3.

*Case 4. B*y considering , for up to first-order MOHAM approximate solutions and two-order OHAM approximate solutions can be compared, and plots is presented in Figure4.

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 In the present study, the prey an predator problem has been solved by OHAM and MOHAM. The results of using these two methods are shown in some plotted in Figures. Comparison with OHAM results higher accurate respect of applying MOHAM, especially for the nods further from the initial nods. Furthermore, MOHAM is reliable and effective for to obtain approximate solutions of the system of nonlinear volterra differential equations

**REFERENCES**

[1] N. R.Anakira, A. K. Alomari, A. F. Jameel, I. Hashim, “Multistage Optimal homotopy asymptotic method for solving

 initial- value problems,, Journal of Nonlinear Science and Applications. vol. 9, pp. 1826-1843, 2016.

[2] J. Biazar, R. Montazeri, “Optimal homotopy asymptotic and multistage optimal homotopy asy- mptotic methods for Ab-

 el Volterra integral equation of the second kind. Computational Methods for Differential Equations, vol.8, no. 4, pp. 770

 -780, 2020.

**[**3**]** J**.** Biazar, and R. Montazeri, “A computational method for solution of the prey and predator problem,, Appl. math. com-

 put.vol. 173, pp. 486-491.

 [4] M.S. H. Chowdhury, I. Hashim, R. Roslan, “simulation of the predator-pey problem by the homotopy -pertubation met-

 hod revised,, Journal of the Juliusz schaauder center, vol. 31, pp.263-270, 2008.