Inverse function for multi-exponential equation

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***Abstract*—Le Bihan et al. demonstrated that the separation of two different anatomical compartments was feasible in the voxel: an intracellular compartment in which molecular diffusion can be assessed, and an extracellular compartment in which the spins’ motions present higher velocities [2]. According to IVIM theory, diffusion is considered bi-compartmental with a fast component of diffusion, related to microcirculation, called pseudo-diffusion, and a slow component of diffusion, linked to pure molecular diffusion. The signal attenuation as a function of** $b$ **is modeled according to a bi-exponential equation:** $SI\_{b}= SI\_{0}.[PF.exp(-b.D\_{fast}) + (1 - PF).exp(-b.D\_{slow})]$ **, where** $PF$ **(also called** $f$**) represents the fraction of the pseudo-diffusion compartment,**$ D\_{fast}$ **(also called** $D^{\*}$**) is the pseudo-diffusion coefficient representing the incoherent microcirculation within the voxel (perfusion-related diffusion) and** $D\_{slow}$ **(also called** $D$**) is the diffusion coefficient representing the slow (pure) molecular diffusion. In addition, the expression (**$1-PF)$ **represents the fraction of the molecular diffusion compartment. Recently, a lot of studies have been done in this field. Diffusion-weighted imaging (DWI) is an important functional imaging technique in oncology, where signal intensity modulated by the (diffusive) motion of water molecules can be used to inform on tumor cellularity, tortuosity of extracellular space, and microstructural organization. While the apparent diffusion coefficient (ADC), conventionally derived by a two-point measurement with application of diffusion-sensitizing magnetic field gradients of varying strengths (*b*-values), has shown utility in oncology for disease localization, diagnosis, staging and assessing therapy response. For further reading, we refer the readers to reference [1], [3], [4] and [5].**

**According to the introductions mentioned so far, we are going to use mathematical modeling, to be able to use mathematical calculations to strengthen the above mathematical relations. Drug concentration is amongst the most important determinants of clinical response to a drug. Variability in**[**pharmacokinetic**](https://www.sciencedirect.com/topics/biochemistry-genetics-and-molecular-biology/pharmacokinetics)**profiles makes drug concentrations unpredictable: the greater the variability, the greater the magnitude of this problem. In fact, we intend to find the inverse function of** $SI\_{b}$**, so that we can calculate the time of drug concentration.**

**In mathematical language, the problem we have to solve is as follows:**$ S\left(t\right)=\sum\_{i=1}^{n}f\_{i}e^{-a\_{i}t}$**, when** $a\_{i,},f\_{i} $**are constants. In fact, our goal is to find the inverse function for this function Or find numerical methods that can find approximate solutions of the inverse function of this function. In this case, by finding the values ​​of time, we will reach our goal. If we set** $ B=\frac{\sum\_{i=1}^{n}f\_{i}}{\sum\_{i=1}^{n}\frac{f\_{i}}{a\_{i}}}$ **, then we have** $S\left(t\right)\~S\left(o\right)e^{-Bt}.$ **Therefore, this method allows us to solve the main problem using numerical methods.**

***Keywords—*IVIM theory*,* Bi-exponential equation *,* Pseudo-diffusion, molecular diffusion, separation.**

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