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Constant-roll inflation in the deformed phase space scenario

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Abstract. This paper's primary purpose of this study is to investigate the constant-roll inflationary scenario with anisotropic conditions concerning the Einstein-æther Scalar-tensor Cosmology in noncommutative phase space. Hence, we present the point-like Lagrangian, which represents the field equations of the Einstein-æther Scalar-tensor model. According to the constant-roll conditions, we take the anisotropic constant-roll inflationary scenario in noncommutative phase space and calculate some cosmological parameters of the mentioned model, such as the Hubble parameter, potential, etc.

Keywords: constant-roll inflationary scenario, Noncommutative parameter

1 Introduction

One of the most famous problems researchers try to solve with different theories is quantum gravity. A common feature that can be found in all topics related to quantum gravity is called the Lorentz violation [1]. However, various gravitational models facing the Lorentz violation have recently received much attention [2–3]. One of the theories of gravity that has received much attention in connection with Lorentz's violate is called the Einstein-æther theory [4]. There exist quantities of the unitary time-like vector field, the æther field. In the Einstein-Hilbert action Integral Selecting, this field violates Lorentz symmetry in the preferred frame. The limitation of Einstein's General Relativity lives while the preservation of locality and covariance formulation is guaranteed in Einstein-æther theory. The mentioned theory, Einstein-æther theory, is called a second-order theory, used to describe different gravitational systems [5]. Einstein-æther theory has many features, and the cosmological applications of this theory have been widely discussed in the literature, including the description of the classical limit of Horava-Lifshitz. Of course, the critical point here is that the opposite is impossible [6]. Of course, from another point of view, scalar fields play a vital role in describing the universe. The field used to describe the early acceleration era of the universe is known as the inflaton field. In addition, scalar fields play a significant role in explaining the late-time acceleration as the solution to the dark energy problem. This paper aims to present a new challenge that has not been explored, namely the constant-roll inflationary scenario for the Einstein-aether scalar-tensor model in the noncommutative phase space, and compare the results with other theories mentioned in the literature. One of the theories that have led to the most important challenges in cosmology so far is called Einstein's theory of general relativity, which has undergone generalizations and modifications [7]. Among these modifications, which have exciting features and results, are modifications of the renormalizability of quantum field theory, which somehow encounters a particular framework that we discuss in this article, called noncommutative space-time. Noncommutative phase space has been addressed in various theories of cosmology, and its various cosmological applications to different theories and frameworks have been discussed. The results have been compared with the latest observable data. You can see for further study in [8]. In many calculations, the effects of noncommutative parameters have been studied in various types of cosmological theories such as power-law inflation, measurement of CMB, etc. [9]. Researchers have recently studied the effects of noncommutative parameters on the inflation scenario of constant rolling in the face of various structures such as fermion systems, modified Brans-Dicke cosmology, and other cosmological forms; results are compared with the latest observable data [10]. The constant-roll inflation scenario has also been of great interest to researchers recently. In this theory, instead of using slow-rolling formalism for inflationary studies, they use a particular condition called the constant-roll condition, which challenges inflation scenarios and provides analytical and accurate answers for some cosmological parameters such as the Hubble parameter, potential, scale factor, and so on. This theory has been widely discussed in the literature. Such a condition has challenged various structures of effective theories such as low energy effective theory, f(R) gravity, and other modified gravitational theories. The results with the latest observational data and accepted and other theories in the literature have yielded exciting results. Some of these works can be found in [11].

The Model

Recently, different types of Einstein-aether cosmological models with the scalar fields have been introduced, and some work has been done in this field in the literature [12]. Among them, the potential of a scalar field for the quintessence field is assumed as a function of specific aether field variables, and its structures are challenged, which has been studied as a general and basic model. Kanno and Soda [13] with the introduction of specific Lagrange, attracted the attention of the scientific community. They introduced an integral action concerning the Einstein-aether coupling parameters, a scalar field function. Such a study led to the fact that this cosmic model experiences two periods of inflation. When the scalar field is dominant, we will have a slow-roll era, and when the aether field contributes to the cosmological fluid, the Lorentz violating state will be established. With respect to [13] the results were presented, including the dynamics of the chaotic inflationary model. The interpretations were used to introduce toy models to study structures such as the Lorentz violating DGP model with no ghosts [14]. The above model has been extensively studied in the literature by researchers, and its various cosmological applications have been investigated, among which you can see in [14]. The Lorentz violating study has also been studied to analyze cosmological histories and cosmological observations and found that Einstein-aether cosmology can be used to describe cosmological observations [15]. Meanwhile, the study of the dynamics models by aether field has been the subject of work of many researchers, and a lot of work has been done that for further research you can see [16,17,18,19,20]. Some researchers have also studied Einstein-aether scalar field cosmology using exact symmetry analysis. Among the most important work done by researchers in recent years, that have quantized in Einstein-aether scalar field cosmology. Using the descriptions detailed in [13] It is discussed that the generalization of the gravitational model can be considered and a scalar field assumed in the Jordan framework, i.e., a scalar field that is coupled with the gravitational section. The Einstein-aether scalar-tensor gravitational model can be considered an integral action as $S = S_{ST} + S_{aether}$, Where difined as;

$$S_{ST} = \int d^4x \sqrt{-g} \, [\, F(\phi)R + g^{\mu\nu}\phi_{;\mu}\phi_{;\nu}/2 + V(\phi) \,] \quad (1)$$

$$S_{aether} = -\int d^4x \sqrt{-g} \, [\, \beta_1(\phi)U^{\nu;\mu}U_{\nu;\mu} + \beta_2(\phi)(g^{\mu\nu}U_{\mu;\nu})^2 + \beta_3(\phi)U^{\nu;\mu}U_{\mu;\nu} + \beta_4(\phi)U^{\mu}U^{\nu}U_{;\mu}U_{\nu} - \lambda(U^{\mu}U_{\nu} + 1)] \quad (2)$$

In the above equation, we have a series of parameters such as β_1, β_2, β_3, and β_4 coefficient functions which describe the coupling between the aether field and the scalar field, and (λ) Lagrange multiplier, which specifies the unitarian of the aether field ($U^{\mu}U_{\mu} + 1 = 0$). Also, the homogeneous and isotropic universe is described using the FLRW flat metric according to the cosmological structures, which is mentioned as $ds^2 = -N^2(t)dt^2 + a^2(t)[dx^2+dy^2+dz^2]$, Here, N(t) and a(t) specify the lapse function and scale factor, respectively, which can explain the three-dimensional radius of the Euclidean space.
According to the above explanations and equations, the Lagrangian point-like equation of the mentioned model, which can describe the field equations, is written in the following form;

$$L(N, a, da/dt, \phi, d\phi/dt) = 1/N\, [\, 6A(\phi)a(da/dt)^2 + 6B(\phi)a^2(da/dt)^2\, d\phi/dt + 1/2\, a^3 (d\phi/dt)^2 \,] - N a^3 V(\phi) \quad (3)$$

3 Noncommutative phase space

Since Einstein's theory of gravitation is not suitable for explaining the structures of the universe at very high energies, researchers have proposed alternatives that result from the modification or expression of theories with new systems. In this regard, various theories and formulations can be mentioned, including the structure Snyder [7,8] Which describes a specific set of the NC spacetime coordinates. This structure introduces a short-length cutoff called the noncommutative parameter, which can modify features such as renormalizability properties of relativistic quantum field theory. [7,8]. NC effects can be significant when dealing with scales where the effects of quantum gravity are substantial. Since the problem of cosmological inflation at such energy scales has significant challenges, the use of such deformed phase space scenarios at these scales seems appropriate to study such a dynamic phase of the universe. In general, such a structure of spacetime has recently been considered by researchers, and many of its cosmological applications have been studied and compared with the latest observable data as well as other works in the literature. For example, clearly in [21] NC spacetime affected on power-law inflation has been investigated and showed a specific function of running the spectra index. We obtain here a specific type of these achievements, which is a particular type of canonical noncommutativity using an appropriate deformation on the classical phase space variables. One of the most important relations that we will benefit from in the calculations is the deformed Poisson bracket between the canonical conjugate moment, expressed as $\{P_a, P_\phi\} = \theta\phi^3$. We can also apply the following formulas in our calculations. $\{Pa, f(P_a, P_\phi)\} = \theta\phi^3\, \partial f\, \partial P_\phi$ and $\{P_\varphi, f(P_a, P_\varphi)\} = -\theta\varphi^3\, \partial f\, \partial P_a$. By tending the parameter (θ) to zero, all the above equations are recovered to standard commutative equations. From this relation, the Hamiltonian is calculated as follows;

$$H = 1/N\, [\, P^2_\phi/2a^3 + 846a^5 (A - 3B2)^3/(aP_a - 6B\, P_\phi)^2 + Na^3V(\varphi) \qquad (4)$$

we calculate the equations as follows;

$$da/dt = \{a, H\} = 1/N\, [\, 864a^5 (A - 3B^2)^3(2\, aP_a - 12\, aB\, P_\varphi)^{-1}] \qquad (5)$$

$$d\phi/dt = 1/N\, [\, P_\phi\, a^3 + 864a^5 (A - 3B^2)^3(-12\, aB\, P_\varphi + 36B)^{-1}] \qquad (6)$$

$$dP_a/dt = 1/N\, [3P^2_\phi/2a^4 + \theta\phi^3/a^3\, P_\phi - 864(A - 3B^2)^3 5a^4/(aP_a - 6BP_\phi) + 864a^5 (A - 3B^2)^3(-2aP_a^2 + 12\, P_a P_\varphi - 12aB\, P_a\, \theta\varphi^3 + 36B\, \theta\varphi^3)^{-1}] \qquad (7)$$

$$dP_\phi/dt = 1/N\, [\, 864a^5/(aP_a - 6BP_\phi)^2 - 3A^2A' + 18A^2BB' - 108AB^3B' - 27A'B^4 + 162B^5B' + 864a^5(A - 3B^2)^3(-2a2\, \theta\varphi^3 P_a + 12aB'P_aP_\varphi + 12aBP_\varphi\theta\varphi^3 - 36B'\, P_\varphi)^{-1}] + Na^3V'(\varphi) \qquad (8)$$

Now according to the above equations, and $dPN/dt = P^2_\phi/2a^3N^2 + (846a^5 (A - 3B^2)^3 / (aP_a - 6BP_\varphi)^2N^2) - a^3V(\phi)$ with respect to N=1, we can calculate the Einstein-æther scalar-tensor cosmology equations in the non-commutativity phase space in the following form;

$$3[(da/dt)/a]^2 = 8\pi G(d\phi/dt/2 + V) \qquad (9)$$

$$[(d^2a/dt^2)/a] + 2[(da/dt)/a]^2 = -\, 8\pi G[d\phi/dt/2 - V + 216(A - 3B^2)^3\, \theta\varphi^3/(A^2A' + B^2B')] \qquad (10)$$

we use equations (9) and (10); in this case, we will have;

$$[(d^2a/dt^2)/a] - [(da/dt)/a]^2 = -(d\phi/dt)^2 - 216(A - 3B^3)^2\, \theta\phi^3/(A^2A' + B^2B') \qquad (11)$$

Here we use the Hubble parameter definition $H = (da/dt)/a$ and rewrite equation (11) as follows;

$$dH/dt = -(d\phi/dt)^2 - 216(A - 3B^2)^3\, \theta\phi^3/(A^2A' + B^2B') \qquad (12)$$

We can calculate the parameter $d\varphi/dt$ as follows using a straightforward calculation with respect to $dH/dt = (dH/d\phi)d\phi/dt$;

$$d\phi/dt = -\, 1/2\, dH/d\phi \pm 1/2\, [(dH\, d\phi)^2 - 864(A - 3B^2)^3\, \theta\phi^3/(A^2A' + B^2B')]^{1/2} \qquad (13)$$

We derivative from the above equation, and the relation used in the constant-roll inflation scenario such as $(d^2\phi/dt^2) = -(3 + \alpha)H\phi$, we will have;

$$-(3 + \alpha)H = -\, 1/2\, d^2H\, d\phi^2 \pm [\, d^2H\, d\phi^2\, dH/d\phi - ((864[\, 3A^2A' - 18AA'B^2 - 18BB'A^2 + 27B^4 + 108AB'B^3 - 162B'B^5]\, \theta\phi^3 \times 3\theta\phi^2 (A - 3B^2)^3) \times (A^2A' + B^2B') - 864(A - 3B^2)^3\, \theta\phi^3 (A''A^2 + 2AA'^2 + 2BB'^2 + B^2B''))/(2[(dH\, d\phi)^2 - 864(A - 3B^2)^3\, \theta\phi^3/(A^2A' + B^2B')]^{1/2}] \qquad (14)$$

if we consider parameters such as (θ), A, and B as zero, equation (14) becomes an ordinary equation in the literature. In fact, in this case, equation (14) becomes two equations, one zero and the other becomes the following form. $d^2H/d\varphi^2 - (3 + \alpha)H = 0$. The above equation is an ordinary differential equation whose answer will be calculated as follows;

$$H = c_1 \exp(\sqrt{3 + \alpha}) + c_2 \exp(-\sqrt{3 + \alpha}) \qquad (15)$$

Now we have to assume a particular ansatz to solve the whole equation (22) that contains essential parameters such as (θ), A, and B so that the consequence of these parameters on significant quantities and parameters such as Hubble parameter and potential can be investigated. Hence we will have;

$$H = c_1 \exp(\lambda(\theta, A, B)\sqrt{3+\alpha}) + c_2 \exp(-\lambda(\theta, A, B)\sqrt{3+\alpha}) \quad (16)$$

The parameter (λ) can be calculated directly by placing (25) in the equation (22). By calculating this parameter (λ) and placing it in equation (25), we can calculate the explicit relationship for the Hubble parameter according to different boundary conditions. Then we can use it to calculate other quantities such as Hubble parameter. In this way, we can have the Hubble parameter by creating the appropriate boundary conditions. By calculating the Hubble parameter, all other important quantities such as potential، scale factor and velocity field ($d\phi/dt$) and other quantities considered in the inflation scenario can be quickly investigated. Therefore, in the continuation of the article, we assume two boundary conditions and calculate the Hubble parameter appropriate to each boundary condition. Finally, we summarize the results of our work. Due to the large of potential sentences V, etc., their calculation is ignored. Therefore, we know that other desired quantities can be quickly and directly calculated by investigating a Hubble parameter. we apply the first boundary condition ($c_1 = c_2 = M/2$) to equation (16). In the following, different answers are obtained for the Hubble parameter, in which we consider only the positive solution. Therefore, according to the concepts mentioned earlier, the Hubble parameter is calculated according to the first boundary condition in the following form;

$X = 3\sqrt{3}\,[\,2(3+\alpha)^{3/2}(-3+\sqrt{3}\phi+\sqrt{3+\alpha})^2 (A''A^2 + B^2B'' + 2AA'^2 + 2BB'^2)/(27\sqrt{3}B^4B' + AA'' + 2AA'B^2 + 64AB''B^3) - \phi^2(3A^2A' + 18AA'B^2\phi - 162B'B^5\phi + 9B'(-2+3B'+2B'\phi))\theta/(A^2A' + B^2B')^2 + 864(A-3B^2)^3 \theta\phi^3(2AA'^2 + B(2B'^2 + BB')) + A^2A'/(A^2A' + B^2B')^4\,]$,

$Y = 2(M^2\phi^2(3+\alpha)^2(-6-\alpha+2\sqrt{3}\sqrt{3+\alpha})$,

$$H = M\cosh[\,\phi\,(M(3+\alpha) - X/Y)/\,2\sqrt{3}(3+\alpha)(M + M\sqrt{3+\alpha}/\sqrt{3})]. \quad (17)$$

Now, using the above equation, we can calculate the potential for the first case, and another important parameter, i.e., ($d\varphi/dt$). other quantities such as scale factor can be calculated according to the definition of the Hubble parameter. In the continuation of this article, we apply the same calculations for the second boundary condition. We apply the second boundary condition ($c_1 = M/2$, $c_2 = -M/2$) to equation (16). Hence the Hubble parameter is calculated according to the second boundary condition as follow;

$O = 27\theta\phi[\,(A^2A' + B^2B')(162A^2 + 18AA'\phi - 6AB^2 + 9B'(-2AA' - 12ABB' + 3B' + 36B'B^5)))- 864(A + 3B^2)^3\phi + 2B'\phi(2AA'^2 + B(2B'^2 + BB') + A^2A'') + (A^2A'' + 18AA' + B^2B''\phi)\times(-3AA'(-3+\alpha)\phi)$,

$S = M\sqrt{3+\alpha}\sqrt{M^2(3+\alpha)(-3+\phi^2(3+\alpha))}(A^2A' + B^2B')^4$,

$$H = M\sinh[\phi\,((3+\alpha) + \sqrt{162M/\sqrt{M^2(3+\alpha)(-3+\phi^2(3+\alpha)^2)}} - O/S)/\,12\sqrt{3}(3+\alpha)]. \quad (18)$$

Also, according to the previous subsection, each of the parameters and quantities such as potential and $d\varphi/dt$ and other quantities such as scale factor for this case can be calculated.

Concluding remarks

In this paper, the primary purpose of this study was to investigate the constant-roll inflationary scenario with anisotropic conditions concerning the Einstein–æther Scalar-tensor Cosmology in noncommutative phase space. That is, we first introduced an Einstein–æther scalar-tensor cosmological model. In this structure, in action integral of scalar-tensor, one is introduced æther field with æther coefficients that it be a function of the scalar field, which is, in fact, a kind of extender of the previous Lorentz-violating theories. Hence, we presented the point-like Lagrangian, which represents the equations of the Einstein–æther Scalar-tensor model. Then we calculated the Hamiltonian of our model directly. According to the noncommutative phase space characteristics, we obtained the some equations of this model. In the following, according to the constant-roll conditions, we studied the anisotropic constant-roll inflationary scenario and calculated some cosmological parameters of the mentioned model, such as the Hubble parameter, potential, etc. The findings of the mentioned paper can be extended to other scalar-tensor theories and challenge their cosmological applications. The model mentioned in this paper or other models can also examine the scalar-tensor cosmology in different contexts and compare the results. The findings of this article can be challenged with a new idea that has recently been very much of interest to researchers, namely to the swampland program, and compare the results with the latest observable data. It is also possible to study different theories of cosmology in the noncommutative phase space and select the best models among them that are most consistent with the latest observable data. Also, examining different types of cosmological models with such a proposed structure provided in this paper can propose a new classification for cosmological models.

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