**Dirac particle near R - N black hole, quasinormal mode and harmonic oscillator**

**energy**

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**Abstract.** In this paper, we try to study the behavior of the Dirac particle in the background

of the Rosener Nordstrom black hole and try to obtain its energy spectrum and we will examine it. Also use energy spectrum to obtain the entropy and particle information close to the event horizon.

Keywords: Dirac equation, Entropy, Energy spectrum, R - N black hole

**1 Introduction**

One of the successes of general relativity is the mysterious physical prediction of the black

hole in the universe, for this reason, theoretical physics has been studying the black hole for

decades. Also, black holes are one of the quantum phenomena due to Hawking radiation.

So black holes can help us understand the relationship between general relativity theory and

quantum mechanics. We consider the theory of quantum fields in curved space-time and the

behavior of different fields interacting with the gravitational field to construct a theory that

incorporates both quantum mechanics and general relativity [1, 2]. That's why black holes

are being studied to obtain information and its implications are mentioned in these articles

[3, 4, 5]. Black holes have three observable quantities, mass, electric charge and angular

momentum, where are divided into different categories. The black hole with a charge and

mass is called a Reissner-Nordstrom black hole, similar to the Schwarzschild black hole but

with two outer and inner event horizons. So, in the limit (Q 0) we will have the same

Schwarzschild black hole also in the limit of (Q M) will have extreme Reissner-Nordstrom

black hole. When we solve a scalar or Dirac field in the curve space, get to imaginary energy

that is first time the term quasi normal mode used by Simone and Will and Fiziev [6, 7],

which is an interesting property and plays an important role. He plays in black hole physics.

The quasinormal state for black holes also produces a set of discrete frequencies and is made

up of two real and imaginary parts, we whose real part corresponds to the actual frequency

and its imaginary part is proportional to the attenuation rate when (> 0). It decreases

from the oscillator, and when it is ( < 0) it indicates instability. The quasinormal

modes provide a unique opportunity for black hole recognition. Hopefully in the near future

large-scale interferometric detectors will be exploited to detect gravitational waves. In order

to extract as much information as possible from the gravitational wave signal it is important

that we achieve how the quasinormal modes depend on the black hole parameters. According

to the article [8] we considered the Klein-Gordon particles near the Reissner-Nordstrom black

hole and obtained its entropy and also showed that each particle contains 8 bits of information stored on the black hole. Now in this paper, we want to investigate the Dirac particles near the Reissner-Nordstrom black hole and investigated entropy and thermal property black hole. All above information give us motivation to investigate following section: In section 2, we introduce the Reissner-Nordstrom black hole. In section 3,write the Dirac equation in curve space-time. In section 4, we employ the particle on Reissner-Nordstrom black hole and obtain the energy and wave function in near Reissner-Nordstrom black hole. In section 5, we calculate thermal properties of Reissner-Nordstrom black hole with energy spectrum and finally, we explain the result in section 6.

**Reissner-Nordstrom black hole**

The Reissner-Nordstrom black hole is described by two observable quantities Q and M, the

metric being written as follows [9]

*ds2 = f(r)dt2 - dr2 - r2dθ2 - r2sin2dφ2* (1)

This is achieved by placing f(r) = 0, two outer and inner event horizons for the black hole, where f(r) can be written in terms of outer and inner  *= M ±*  event horizons

 (2)

**Dirac equation in curve space-time**

As we know, the Dirac equation of spin particles are invariant under the Lorentz transformation. So in Minkoweski at space-time, we have the following equation

 (3)

where is the standard Dirac matrix 4×4 in space-time and is shown below in terms of Pauli , i = 1, 2, 3 and unit matrices

Now, we use the covariant derivative and the tetrad representation, the Dirac equation in curved space-time will be following

 (4)

where and are at and curved tensor space-time respectively. Thus, the Dirac equation

in curved space-time with electromagnetism can be written by

 (5)

and

 (6)

Also, and can be obtained as follows,

 - (7)

where is Christofel symbol.

**Energy spectrum of particle near the proximity Reissner-**

**Nordstrom black hole**

According equations also, when (Q is charged black hole) and Consider the wave function as , Dirac equation is obtained as following

 (8)

For the spherical part defamation operator K [12]:

 (9)

where is eigenvalues of operator K. So, the spherical part and will be following

 (10)

where in this case, separated we get two separate equations for the radial part

by Considering this equation near the outer event horizon, and taking into account

 , one can calculate

 (11)

 (12)

When we solve the equation for the massless Dirac particle m = 0, we get following solutions

[65],

 (13)

Also by using we have

 (14)

and

 (15)

where

 (16)

We choose the wave function that decreases at a constant damping rate, so the energy spectrum (14) is considered as particle energy near the R-N black hole. Then

use energy to obtain the entropy and thermal properties of the black hole.

**Thermal properties of Reissner-Nordstrom black hole**

**with energy spectrum**

We consider N particles near Reissner-Nordstrom black hole also use energy particle and

partition function to obtain entropy and thermal properties of the black hole. So,we compute

the partition function for N indefinite particles in the canonical set as follows [14]:

 (17)

and use of energy spectrum (14) to obtain Q:

 (18)

where is invert temperature . We chose real part of and

note that the average energy and use Eqs. (17) and (18), we will arrive at following average value of energy:

 (19)

We can also obtain the heat capacity directly using the average energy by the following

formula:

 (20)

As can be seen in the above equation, at high temperatures the heat capacity is equal to the

constant number of particles. We obtain the entropy using the average energy and the

partition function as follows [14]:

 (21)

Now, we calculated entropy with consider the following Stirling's approximation

(ln N!=N ln N-N) and use Eqs. (17), (18), (19) and (21), so we have the following

 (22)

**Holography and information theory**

As we know the boundary of curve as R-N black hole play important role in holography

and AdS/CFT. Here, also one can think about the boundary of R-N black hole as a storage

device for information. Assuming that the holographic principle holds, the maximal storage

space, or total number of bits, is proportional to the area A. Let us denote the number of

used bits by N0, it is natural to assume that this number will be proportional to the area

[15]. We know from the holographic principle that all information about an object is stored

on its surface. It also states that a bit of information is stored in an area the length of

Planck. For the black hole, its information is stored on its boundary and we use the event

horizon area to describe its feature. In this section, we want to find how particles are stored

near the black hole of the R - N black hole event on the surface of the black hole event

horizon. First, we obtain the maximum entropy in N of Eq.(22), as follows:

 (23)

Given the particles near the event horizon, we can consider the number of particles proportional to the surface of the black hole

 (24)

where q is a constant value. Also, we can set the maximum entropy equal to the Bekenstein

entropy and [16, 17]. In this case, according to the Eqs. (23) and

(24), q = 4 is obtained. According to Verlinde's theory [15], each surface can be considered

as information bits, so we consider the surface of the black hole as a set of information bits

and write its relation as follows:

 (25)

Given the Eqs.(24) and (25), we obtain the relation of the particle number to the information

bits as follows:

 (26)

It shows that each particle in near R - N black hole has 4 bit of information.

**discussion and result**

In this article, we examined the behavior of the Dirac particle and , the behavior of the

Klein Gordon particle in the background of the Klein Gordon black hole in Article [8]. We

came to this conclusion by comparing these two articles. For Klein Gordon particles, the

energy obtained was real and in the form of a harmonic oscillator, but for Dirac particles,

the energy was real and imaginary, and the real part was in the form of a harmonic oscillator

plus an additional sentence.

**Conclusions**

We used holography and found that each Klein Gordon particle contained eight

bits of information on the surface of the Nordstrom Resener black hole, while the Dirac

particle contained four bits of information. One of the factors that makes Dirac particle less

information than Klein Gordon particle wind can be the principle of Paula's exclusion.

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