Study of the Penrose process at WGC condition for the charge rotating BTZ black hole

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Abstract

In this paper, we study the collision of spinning particles near a charged rotating BTZ black hole with WGC condition, and we obtain the extracted energy of the black hole by the Penrose process. We assume two particles fall from infinity and collide near a black hole. During this collision, one particle falls into the black hole, and the other escapes to infinity. We examine and calculate the maximum efficiency parameter (η). We mention that by exerting the weak gravity conjecture, we have $\eta_{max} \simeq 1250$. On the other hand, the particles created in the collision have a specific range of spin, which can lead to the formation of unknown particles. This result can enhance our understanding of how black holes work, how they die, and better study physical, astronomical black holes. A thorough understanding of black holes helps us understand how a holographic system works.
Keywords: Weak Gravity Conjecture, Penrose Process, BSW Method, Block Holes.

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Contents

1 Introduction

Several researchers have studied particle collisions in both spinning and non-spinning particles near the background of black holes, and it has a long history. The black hole is the most strong system globally, and the energy extracted from the black hole is much more efficient than nuclear energy. Of course, this can be extracted from certain black holes. Penrose suggested rotating black holes for energy extraction. The possibility of a particles collision near a black hole was first investigated by Piran and others in 1975 [1]. Also, in the continuation of this path, a beautiful event took place and discovered a new particle collision mechanism by Banãdos, Silk and West (BSW), in 2009 [2]. As a result, due to the increased energy of the particles in collision, rotating black holes are considered an accelerator [2, 3]. In recent years, different black holes have studied various aspects of the BSW mechanism. Many results have been investigated, such as Kerr naked singularity and rotating black holes and their universal properties. We see Refe.s [4–7]. Also, much research has been done on the various dimensions of space-time (higher or lower), such as the five-dimensional Kerr black hole and three-dimensional charged black holes [8–10]. The Penrose process has a wide range of applications. For example, the BSW mechanism investigates the Penrose collision, leading to the extraction of energy from the black hole through particle collisions, which can help us in the relevant optimizations of the Penrose process. We assume the particles that fall into the ergo region of the black hole are broken into two particles. The broken particle with negative energy sinks into the black hole, and another with positive energy escapes to infinity. The resulting energy extraction from rotating black holes is called the Penrose process. This method is expected to provide a more efficient mechanism in astrophysical conditions. At high energies, unknown particles can form, which indicates new physics. So this motivates us to study rotating black holes because even when the energy is arbitrarily large, the particle can not escape to infinity, and it will be crucial

to examine the maximum efficiency of extracted energy from black holes [11–19].
In this article, we peruse BTZ black Holes. A specific stationary black hole solution was first investigated for three-dimensional space-time with a negative cosmological constant by Banãdos-Teitelboim-Zanelli (BTZ) [20]. Due to the simplicity and similarity of this solution with the (3+1)-dimension Kerr black hole, it has received much attention in recent years. Also, the collision of non-spinning particles is studied around the (2+1)-dimensions black holes by [21–24]. This beautiful model helps to gain a deep understanding of the BTZ background. Many problems are simpler and more analytical in the BTZ space-time than in Kerr [22–24]. In the study of particle collision, many authors usually focus on particle geodetic path, so the motion equations of spinning particles around a space-time background are described by Mathisson-Papapetrou-Dixon(MPD) equations [26–28]. Here, we take advantage of the collisions associated with the Kerr space [25] and investigate the particle collision on the CR BTZ black hole.
Also, the weak gravity conjecture (WGC) plays a significant role in this paper. Most recently, WGC has been studied in various fields, including aspects of this conjecture, such as swampland and landscape. In those cases, we can say the theories set inconsistent and consistent with quantum gravity are swampland and landscape, respectively. The WGC also contains another conjecture, such as Trans-Planckian-Censorship-Conjecture (TCC). Generally, for different applications of WGC, one can see Refe.s [29–60]. As we know, black holes have extremal conditions ($Q = M$) in the WGC. This condition is slightly different in rotating black holes covered in the third section. In general, weak gravity conjecture can have new practical aspects. This article wants to examine the energy obtained from the collision of two spinning mass particles from the BSW method at WGC conditions.
The above discussions motivate us to organize the corresponding paper as a following. In section 2, thoroughly examines the collision of spinning particles near the charged rotating BTZ black hole (CR BTZ) to BSW methods. In section 3, we explain the WGC condition for charged and rotating black holes. In section 4, We study two essential constraints that orbits create on particles. In section 5, we examine the process performed in section 2 with the weak gravity conjecture method. Moreover, in the final section, we examine the results obtained from these two methods.

2 The motion equations of spinning particles

One of the extraordinary phenomena to investigate extracted energy is colliding two spinning mass particles near a charged rotating black hole. For this reason, first, we consider the metric of CR BTZ black hole, which is given by

$$
\begin{aligned}
ds^2 &= -f(r)dt^2 + \frac{dr^2}{f(r)} + r^2(d\phi + \frac{j}{2r^2}dt)^2, \\
f(r) &= -M + \frac{r^2}{l^2} + \frac{j^2}{4r^2} - \frac{\pi Q^2 \log(r)}{2}
\end{aligned}
\tag{1}
$$

Here M, Q and j are mass, charge and angular momentum of black hole respectively and $l^2 = (-\frac{3}{\Lambda})$, $8G = c = 1$. According to the space-time background, the equations of motion can be calculated with MPD equations [54, 55].

$$
\begin{aligned}
\frac{DP^a}{D\tau} &= -\frac{1}{2}R^a_{bcd}v^b S^{cd}, \\
\frac{DS^{ab}}{D\tau} &= P^a v^b - P^b v^a.
\end{aligned}
\tag{2}
$$

Here $v^a = (\frac{\partial}{\partial r})^a$, $\frac{D}{D\tau}$, P^a and S^{ab} are tangent vector, covariant derivative, 4-momentum and the spin tensor, respectively. To get logical relations between P^a and v^a, we use the following conditions [54],

$$
\begin{aligned}
S^{ab}P_b &= 0, \\
P^a v_a &= -m.
\end{aligned}
\tag{3}
$$

By combining the Eqs. (2) and (3) one can obtain the following equation,

$$
mv^a - P^a = \frac{S^{ab}R_{bcde}P^c S^{de}}{2(m^2 + \frac{1}{4}R_{bcde}S^{bc}S^{de})}. \tag{4}
$$

Generally, we know that 4-momentum is not parallel to 4-speed of spin particles in $4-$dimensions. Of course, our space-time will be $(2+1)-$dimensions. For two killing vectors in CR BTZ space-time $\zeta^a = (\frac{\partial}{\partial t})^a$ and $\phi^a = (\frac{\partial}{\partial \phi})^a$, we have the following equations,

$$
\begin{aligned}
e^{(0)}_a &= \sqrt{f(r)}(dt)_a, \\
e^{(1)}_a &= \frac{(dr)_a}{\sqrt{f(r)}}, \\
e^{(2)}_a &= r(d\phi)_a - \frac{j}{2r}(dt)_a.
\end{aligned}
\tag{5}
$$

So, the corresponding conserved quantity concerning ζ^a will be as,

$$
Q_\zeta = P^a\zeta_a - \frac{1}{2}S^{ab}\nabla_b\zeta_a. \tag{6}
$$

By using the above equation, we can calculate the two conserved equations, which are energy per unit mass E_m and angular momentum per unit mass J_m and are given by,

$$\begin{aligned} E_m &= -u^a\zeta_a + \frac{1}{2m}S^{ab}\nabla_b\zeta_a, \\ J_m &= u^a\phi_a - \frac{1}{2m}S^{ab}\nabla_b\phi_a. \end{aligned} \tag{7}$$

Using equations (5) and (6) for the CR BTZ metric (1), we can obtain

$$\begin{aligned} E_m &= \sqrt{f(r)}u_0 + (\frac{j}{2r} - \frac{\pi s Q^2}{4r} + \frac{sr}{l^2})u_2, \\ J_m &= s\sqrt{f(r)}u_0 + (-\frac{2r^3}{j} - \frac{j^2 s}{4r^3} - \frac{\pi s Q^2}{4r} + \frac{rs}{l^2} - \frac{2r^3 s}{j-2r^2})u_2. \end{aligned} \tag{8}$$

The above equations help us to obtain the dynamic velocity u as,

$$\begin{aligned} u_0 &= \frac{E_m(-4r - \frac{2js}{r}) + J_m(\frac{2j}{r} - \frac{\pi s Q^2}{r} + \frac{4rs}{l^2})}{(-4r - \frac{\pi s^2 Q^2}{r} + \frac{4rs^2}{l^2})\sqrt{f(r)}}, \\ u_2 &= \frac{J_m - E_m s}{r + (\frac{\pi Q^2}{4r} - \frac{r}{l^2})s^2}, \end{aligned} \tag{9}$$

In order to obtain u_1, we use $(u_0)^2 - (u_1)^2 - (u_2)^2 = m^2$ condition. So, the corresponding u_1 will be as,

$$u_1 = \rho\sqrt{-m^2 - \frac{16(J_m - E_m s)^2}{(-4r - \frac{\pi s^2 Q^2}{r} + \frac{4s^2 r}{l^2})^2} + \frac{(E_m(-4r - \frac{2js}{r}) + J_m(\frac{2j}{r} - \frac{\pi s Q^2}{r} + \frac{4rs}{l^2}))^2}{(-4r - \frac{\pi s^2 Q^2}{r} + \frac{4rs^2}{l^2})^2 f(r)}}. \tag{10}$$

By using the above equations with appropriate replacements, one can obtain v_0, v_1 and v_2 as,

$$\begin{aligned} v_0 &= (-4r - \frac{\pi s^2 Q^2}{r} + \frac{4rs^2}{l^2})u_0, \\ v_1 &= (-4r - \frac{\pi s^2 Q^2}{r} + \frac{4rs^2}{l^2})u_1, \\ v_2 &= (-4r + \frac{\pi s^2 Q^2}{r} + \frac{4rs^2}{l^2})u_2. \end{aligned} \tag{11}$$

The motion equation of spinning particles in the CR BTZ space-time can be calculated according to the above equations.

$$\begin{aligned}
P_0 =& m(\frac{E_m(-4r-\frac{2js}{r})+J_m(\frac{2j}{r}-\frac{\pi s Q^2}{r}+\frac{4rs}{l^2})}{\sqrt{f(r)}}),\\
P_1 =& m\rho[-(4r+\frac{\pi s^2 Q^2}{r}-\frac{4rs^2}{l^2})^2 m^2-16(J_m-E_m s)^2\\
&+\frac{1}{f(r)}(E_m(-4r-\frac{2js}{r})+J_m(\frac{2j}{r}-\frac{\pi s Q^2}{r}+\frac{4rs}{l^2}))^2]^{\frac{1}{2}},\\
P_2 =& 4m(-4r+\frac{\pi s^2 Q^2}{r}+\frac{4rs^2}{l^2})(\frac{J_m-E_m s}{4r+\frac{\pi s^2 Q^2}{r}-\frac{4rs^2}{l^2}}).
\end{aligned} \tag{12}$$

Where $P_0 = P^t(r), P_1 = P^r(r)$, $P_2 = P^{\phi}(r)$ and $\rho = \pm$. We note that the sign $+$ and $-$ indicate the particle direction is outward and inward, respectively.

3 The WGC condition

Generally, one can say that the extreme black holes must be unstable in any corrected theory as a quantum gravity (except in cases we have symmetry). Therefore, there must be states in the following form,

$$(M,Q) \rightarrow (M', Q-q) + (m,q), \qquad M > M', m. \tag{13}$$

As we know, the extreme condition $M = Q$ causes us to consider a state like $\frac{q}{m} \geq 1$. On the other hand, we know that the decay equations for the charged black hole with multiple U(1) are given by,

$$Q^a = \sum_i n_i m_i \xi^i_a, \qquad M > \sum_i n_i m_i, \tag{14}$$

also, we have the following equation,

$$\overline{\xi_i} = (\xi^a_i), \qquad \xi^a_i = \frac{q^a_i}{m_i}. \tag{15}$$

One can rewrite of equations as,

$$Z_a = \sum_i \sigma_i \xi^a_i, \qquad 1 > \sigma_i, \tag{16}$$

where $Z_a = \frac{Q^a}{M}$ and $\sigma_i = \frac{n_i m_i}{M}$. Similarly, in the case of convex shells with multiple charges, the decay equations for a charged rotating black hole is given by,

$$J = \sum_i n_i m_i \xi^i_J, \qquad Q = \sum_i n_i m_i \xi^i_Q, \qquad M > n_i m_i \tag{17}$$

and,

$$\overline{\xi_i} = (\xi_i^J, \xi_i^Q), \qquad \xi_i^J = \frac{J_i}{m_i}, \qquad \xi_i^Q = \frac{q_i}{m_i}. \tag{18}$$

One can rewrite the equations,

$$Z_J = \sum_i \sigma_i \xi_i^J, \qquad Z_Q = \sum_i \sigma_i \xi_i^Q, \qquad 1 > \sum_i \sigma_i \tag{19}$$

here are $Z_J = J/M$, $Z_Q = Q/M$, and $\sigma_i = n_i m_i/M$. The essential point here is that we will have a completely different extreme condition for charged rotating black holes, so in this case, we have,

$$M^2 = Q^2 + \frac{J^2}{M^2}. \tag{20}$$

It can generally consider more configurations such as rotations, charge, Etc. In the appropriate dimensions, we have another condition for a charged rotating black hole,

$$\frac{Z_J^2}{M^2} + Z_Q^2 = 1. \tag{21}$$

We continue our calculations by placing condition WGC in the motion equations of the previous section.

4 Some constraints on the orbits

We consider two particles to check constraints on the orbits; they collide in r with $r \geq r_H$ condition, where r_H is the event horizon radius of a black hole. Particles with a certain angular momentum and energy reach the point of collision in the ergo region of a black hole. We represent the particular value with the impact parameter $b = \frac{J}{E}$. This point is located for critical particles b_c on the event horizon r_H. In order to get such a point, the collision point must be before the horizon. Therefore, our first constraint is obtained with $P_1 \geq 0$ and WGC condition,

$$b = \frac{J_m}{E_m} \geq \frac{4r^2 + 2js}{2j - \pi Q^2 s + 4s(\frac{r}{l})^2}, \tag{22}$$

We can obtain $b_c = \frac{2}{j}$ by the $f(r_h) = f'(r_h) = 0$ conditions. b_c is the critical value, and particles can not reach the horizon under conditions, $b > b_c$. Also, non-critical particles have $b = \frac{2}{j}(1+\gamma)$. Spinning particles must meet the time-like condition to avoid superluminal motions and causality problems. As a result, the second condition is $v^\mu v_\mu < 0$. Using the equations (9), (10), (11) and (9) we obtain,

$$E > \frac{(j^2 s^2 - 4)^2}{16 s (js - 2)}. \tag{23}$$

Moreover, we can specify the range of spins as $s_{min} < s < s_{max}$ by placing $E = 1$. In other words, the time-like condition limits the energy and spin of particles. The values obtained are equal to $s = (-2.17, -0.31, 1, 1.48)$. In fig 1, we plot the energy in terms of spin by equation (22). As you see energy E has only an increasing trend in the range of $-1 < s < 0$.

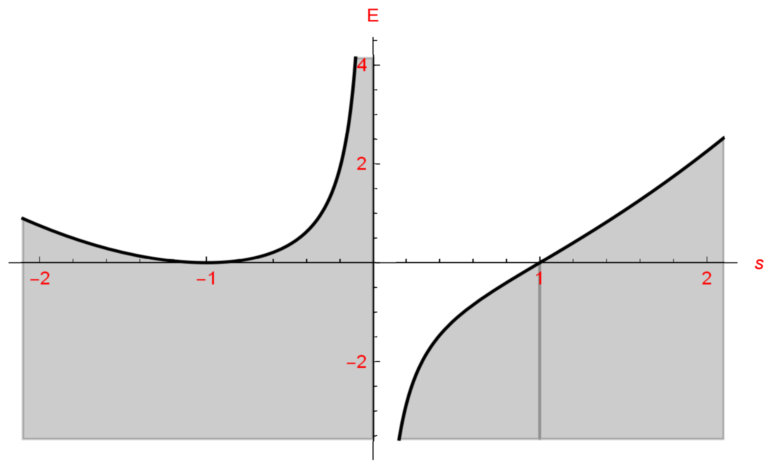


Figure 1: The allowable area indicates the particles' energy E and spins s to reach the event horizon. Particles with the highest energy have a spin of $-0.6 < s < 0$.

5 The Collision of two massive particles

Now we will investigate the collision of two spinning particles near a charged rotating black hole. The two collide before reaching the horizon. One of the particles falls into the black hole and the other escapes to infinity. We expect to have maximum energy efficiency in this case [19]. Particles have the following angular momentum: The first critical particle has $J_1 = \frac{2E_1}{j}$, the second non-critical particle has $J_2 = \frac{2E_2}{j}(1+\gamma)$, and The third near-critical particle has $J_3 = \frac{2E_3}{j}(1 + \alpha\epsilon + \beta\epsilon^2 + ..)$. The last particle has an angular momentum based on the other three particles. We use the laws of energy $E_1 + E_2 = E_3 + E_4$ conservation and angular momentum $J_1 + J_2 = J_3 + J_4$, and obtain the fourth particle's angular momentum $J_4 = \frac{2}{j}(E_1 + E_2(1+\gamma) - E_3(1 + \alpha\epsilon + \beta\epsilon^2))$. Also, the spins conditions and direction are equal,

$$s_2 = s_4, \qquad s_1 = s_3, \qquad \rho_2 = \rho_4 \tag{24}$$

These conditions have a good effect on the super-Penrose process. We assume the mass of particles to be equal $m_1 = m_2 = m_3 = m_4$. Given all these conditions, we calculate the value of P_1 for each particle, and by placing it in equation $P_1^{(1)} + P_2^{(1)} = P_3^{(1)} + P_4^{(1)}$, we can calculate the energy of third particle E_3 based on the first two particles.

$$
\begin{aligned}
E_3 =& \frac{2e_2(\varpi\gamma(1+s_2)^2 - 1 - \varpi + s_2)}{2\varpi^2(2+2s_1+s_1^2+2s_2+s_2^2)} \times \\
& (\pm 2)\sqrt{e_2^2(1+\varpi - s_2 - \varpi\gamma(1+s_2)^2)^2 - X\varpi^2(2+2s_1+s_1^2+2s_2+2s_2)}. \\
X =& (\gamma^2 E_2^2(1+s_2)^2 - E_2^2(2+\gamma-2s_2) - (2E_1E_2(s_2-1) - E_2^2(2+\gamma-2s_2) + \gamma^2E_2^2(1+s_2)^2)) + \\
& 32\rho_1\rho_2(-2E_2^2(s_1-1)^2(E_1^2+(1+s_1)^2)(s_2-1) + \gamma E_2^2(s_1-1)^2(E_1^2+(1+s_1)^2) - \gamma^2 E_2^2 \\
& (s_1-1)^2(E_1^2+(1+s_1)^2)(1+s_2)^2)^{\frac{1}{2}}.
\end{aligned}
\tag{25}
$$

We have $\varpi = (\alpha\epsilon + \beta\epsilon^2 + ...)$ and the following equation obtains the corresponding energy efficiency,

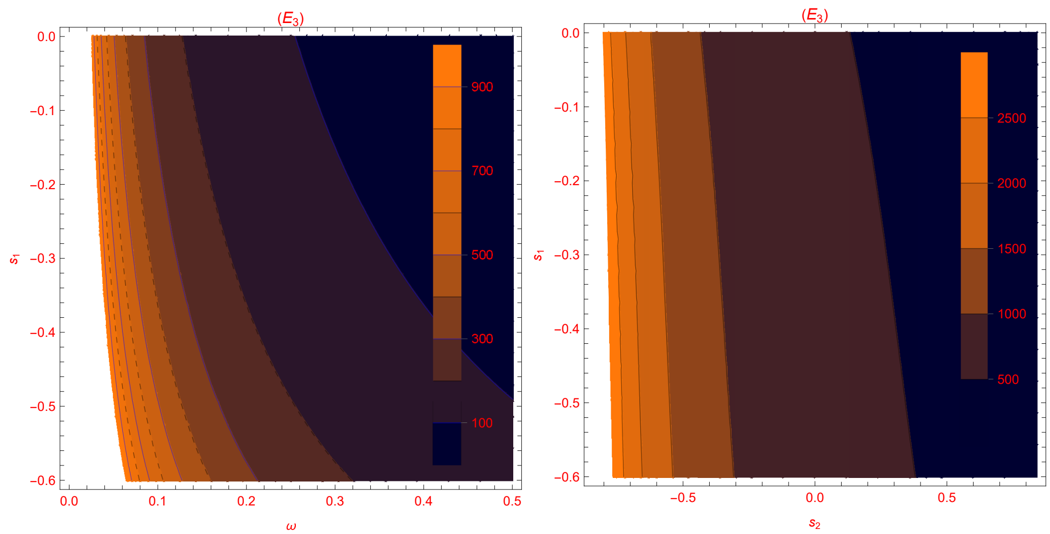
$$
\eta = \frac{E_3}{E_2 + E_1}. \tag{26}
$$

The maximum energy efficiency: We calculated the energy of third particle E3 in terms of E_1, E_2 energy, s spin and some suitable parameters. Two particles are coming from infinity, so we consider $E_1 \geq 1$ and $E_2 \geq 1$. In order to have the best efficiency, we assume that for the first particle, $\rho_1 = +1$, $E_1 \simeq 1$ and the second particle, $\rho_2 (= \rho_4) = -1$, $E_2 \simeq 1$. So, one can obtain The third particle energy as,

$$
E_3 = \frac{-32(s_1-1)\sqrt{-(2+s_1(2+s_1))}}{2\varpi(s_2+1)}. \tag{27}
$$

Now we need to calculate a γ variable for the second particle. We consider it non-critical. So we have $b = \frac{2}{j}(1+\gamma)$ and $E_2 \simeq 1$. By replacing the above-obtained result with the time-like conditions (23), one can obtain $\gamma = (\frac{s_2+1}{s_2})l^2$. In order to acquire E_3 as a straightforward form, we first take s_2 from equation γ and put it into (27). According to Figure 1, we know that energy has an increasing trend in the range of $-1 < s < 0$. So, we have $s_3 = s_1 = -0.3$ and $\varpi = (\alpha\epsilon + \beta\epsilon^2)$. We can plot energy E_3 in terms of s_2 and ϖ, fig (2a).

Fig (2a) shows the highest energy in the lower ϖ. i.e., the third particle has the most energy E_3, while the second particle s_2 is in a near-critical state. Moreover, we see that when we have the maximum energy, the spin s_2 takes on two values, $0.6 \leq s_2 \leq 0$ and is equal to $0.02 \leq \varpi \leq 0.1$. In the last step, we plot the third particle energy E_3 according to the primary particles spin s_2 and s_1 (Fig (2b)). The third particle has the most energy when the primary particles have spin ranges of $-0.6 \leq s_1 \leq 0$ and $-0.78 \leq s_2 \leq -0.5$. These amounts of energy are for when the primary particles have an energy of $(E/m)_1 \simeq (E/m)_2 \simeq 1$, which means



(a) (b)

Figure 2: In (a), we plot the E_3 in terms of ϖ and s_2 whith $\rho_3 = -1$ and $s_1 = s_3 = 0$. In (b), the E_3 in terms of s_1 and s_2, by increasing of s_1, the corresponding energy is increasing.

that black holes can extract much energy in the final moments and extremality. So the values obtained above lead us to have the following equation,

$$\eta = \frac{(E/m)_3}{(E/m)_1 + (E/m)_2} \simeq \frac{2500}{2} \simeq 1250. \tag{28}$$

6 Conclusions

This paper studied the Penrose process for the spinning particles near charged rotating BTZ black holes. Two particles come from infinity and collide near a black hole in this method. One particle falls into a black hole with respect to energy and angular momentum, and the other escapes to infinity. We also calculated the maximum efficiency parameter for the third particle and showed that this value equals $\eta \simeq 1250$. These calculations can help us track physical astrology black holes. It will also enhance our understanding of black holes, how they work, and how they die. Black holes can act as a holographic system, and how they function and die can lead to exciting studies in their holographic structures, such as their thermodynamics and hydrodynamics. We will follow these reviews in future work.

References

[1] T. Piran, J. Shaham, and J. Katz, J. 196, L107 (1975).

[2] M. Ba˜nados, J. Silk and S. M. West, Phys. Rev. Lett. 103, 111102(2009).

[3] M. Ba˜nados, B. Hassanain, J. Silk, and S. M. West, Phys. Rev. D 83, 023004 (2011).

[4] M. Patil and P.S. Joshi, Classical Quantum Gravity 28, 235012 (2011).

[5] S. W. Wei, Y. X. Liu, H. Guo, and C. E. Fu, Phys. Rev. D 82, 103005 (2010).

[6] Y. Li, J. Yang, Y. Li, S. Wei and Y. Liu, Classical Quantum Gravity 28, 225006 (2011).

[7] O. B. Zaslavskii, Phys. Rev. D 82, 083004 (2010).

[8] A. Abdujabbarov, N. Dadhich, B. Ahmedov, and H. Eshkuvatov, Phys. Rev. D 88, 084036(2013).

[9] A. Tursunov, M. Kolos, A. Abdujabbarov, B. Ahmedov, and Z. Stuchlik, Phys. Rev. D 88, 124001 (2013).

[10] J. Sadeghi, B. Pourhassan, and H. Farahani, Communications in Theoretical Physics, Volume 62, Number 3. 12

[11] T. Piran and J. Shaham, Phys. Rev. D 16, 1615 (1977).

[12] M. Bejger, T. Piran, M. Abramowicz, and F. Hakanson, Phys. Rev. Lett. 109, 121101 (2012).

[13] J. D. Schnittman, Phys. Rev. Lett. 113, 261102 (2014).

[14] E. Berti, R. Brito, and V. Cardoso, Phys. Rev. Lett. 114, 251103 (2015).

[15] E. Leiderschneider and T. Piran, Phys. Rev. D 93, 043015 (2016).

[16] K. Ogasawara, T. Harada, and U. Miyamoto, Phys. Rev. D 93, 044054 (2016).

[17] T. Harada, K. Ogasawara, and U. Miyamoto, Phys. Rev. D 94, 024038 (2016).

[18] K. Ogasawara, T. Harada, U. Miyamoto, and T. Igata, Phys. Rev. D 95, 124019 (2017).

[19] K. i. Maeda, K. Okabayashi, and H. Okawa, Phys. Rev. D 98, 064027 (2018).

[20] M. Ba˜nados, C. Teitelboim, and J. Zanelli, Phys. Rev. Lett. 69, 1849(1992).

[21] N. Tsukamoto, K. Ogasawara, Y. Gong, Phys. Rev. D 96, 024042 (2017).

[22] R. B. Mann, J. J. Oh, and M. I. Park, Phys. Rev. D 79, 064005 (2009).

[23] J.V. Rocha and V. Cardoso, Phys. Rev. D 83, 104037(2011).

[24] Kota Ogasawara, and Naoki Tsukamoto, Phys. Rev. 99, 024016 (2019).

[25] M. Guo, S. Gao, Phys. Rev. D 93,084025 (2016).

[26] E. Hackmann, Cl. Lammerzahl, Yu. N. Obukhov, D. Puetzfeld, and I. Schaffer, Phys. Rev. 90, 064035(2014).

[27] M. Saijo, K. Maeda, M. Shibata, and Y. Mino, Phys. Rev. D 58, 064005 (1998).

[28] P.I. Jefremov, O. Y. Tsupko, and G. S. Bisnovatyi-Kogan, Phys. Rev. D 91, 124030 (2015).

[29] E. Palti, Phys. 67, 6, 1900037 (2019).

[30] N. A. Hamed, L. Motl and A. Nicolis, JHEP 0706, 060 (2007).

[31] A. Medved, Mod. Phys. Lett. A, 22, 2605-2610 (2007).

[32] A. Urbano, arxiv: 1810.05621 (2018).

[33] Y. Akrami, R. Kallosh, A. Linde and V. Vardanyan, Fortsch. Phys. 67, no.1-2, 1800075 (2019).

[34] T. Brennan, F.Carta and C. Vafa, PoS TASI2017, 015 (2017).

[35] H. Murayama, M. Yamazaki and T. Yanagida, JHEP 12, 032 (2018).

[36] C. Vafa, arxiv:/hep-th/0509212v2 (2005).

[37] Eran. Palti,JHEP 2017, 34 (2017).

[38] Karta, Kooner, Susha Parameswaran and Ivonne Zavala, Phys. Lett. B 759, 402409 (2016).

[39] Miguel Montero, Gary Shiu and Pablo Soler, JHEB 2016, 159 (2016).

[40] Prashant Saraswat, Phys. Rev. D 95, 025013 (2017).

[41] Stefano Andriolo, Daniel Junghans, Toshifumi Noumi and Gary Shiu, Fortschritte der Physik 66, 1800020 (2018).

[42] Yu Akayama and Yasunori Nomura, Phys. Rev. D 92, 126006 (2015).

[43] J. Sadeghi, S. Noori Gashti and E.Naghd Mezerji. Phys. Dark Univ 30, 100626, doi: 10.1016/j.dark.2020.100626 (2020).

[44] Z. Yi and Y. Gong, arxiv:1811.01625 (2018).

[45] M. Artymowski and I. Ben-Dayan, JCAP 1905, 042 (2019).

[46] S. Brahma and M. Hossain, JHEP 2019, 6 (2019).

[47] David Andriot, Physics Letters B, 785:570573 (2018).

[48] Gia Dvali and Cesar Gomez, Fortschritte der Physik, 67(1-2):1800092 (2019).

[49] J Sadeghi, B Pourhassan, S Noori Gashti, E Naghd Mezerji, A Pasqua, arXiv:2108.01448 (2021).

[50] J Sadeghi, S Noori Gashti, E Naghd Mezerji, B Pourhassan, arXiv:2011.05109 (2020).

[51] David Andriot and Christoph Roupec, Fortsch. Phys. 67(1-2):1800105 (2019).

[52] Sumit K Garg and Chethan Krishnan, arXiv:1807.05193 (2018).

[53] Hirosi Ooguri, Eran Palti, Gary Shiu, and Cumrun Vafa, Physics Letters B, 788:180184 (2019).

[54] Tom Rudelius, arXiv:1905.05198, (2019).

[55] Jerome Martin and Robert H Brandenberger, Physical Review D, 63(12):123501, (2001).

[56] R. M. Wald, Gravitational spin interaction, Phys. Rev. D 6, 406 (1972).

[57] J Sadeghi, E Naghd Mezerji, S Noori Gashti, Logarithmic corrected gravitational model and swampland conjectures, arXiv:1910.11676 (2019).

[58] J Sadeghi, EN Mezerji, SN Gashti, Modern Physics Letters A, (2021).

[59] J Sadeghi, B Pourhassan, S Noori Gashti, S Upadhyay, E Naghd Mezerji, arXiv:2011.14366 (2020).

[60] M. Guo, S. Gao, Kerr black holes as accelerators of spinning test particles, Phys. Rev. D93,084025 (2016).