Thermodynamic Phase Transition of Generalized Ayon-Beato Garcia Black Holes with Schwarzschild anti de Sitter space time perspective.

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Abstract

In this work we study thermodynamics of generalized Ayon-Beato and Garcia (ABG) black hole metric which contains three parameters named as mass m, magnetic charge q and dimensionless coupling constant of nonlinear electrodynamics interacting field γ. This is done at extended phase space where we need a cosmological parameter which behaves as the pressure thermodynamic coordinate. We generate the necessary cosmological parameter from the charge parameter of the ABG metric field. In short we first extract a variable cosmological parameter together with a variable mass function such that the ABG black hole metric can be shown similar to a Schwarzschild anti de Sitter form apparently. Then by calculating the Hawking temperature of the black hole we obtain equation of state. By studying isothermal P-v curves we infer that the system participates in the Hawking-Page phase transition where the disequilibrium evaporating ABG black hole reaches finally to a vacuum AdS space. Other diagrams such that Gibbs free energy, heat capacity and entropy satisfy possibility of phase transition and there is also a coexistence point in phase space depended to γ value where the two different phases exist synchronously. For small scale black holes there are three phase while for larger than there are just two phase.

1 Introduction

The Einstein general theory of relativity is the most efficient theory of the gravity, which its validity has been approved through its correspondence to the observations and experiments in many years [1]. But in some cases, this theory is not practical. For instance the spacetime singularity, which is an example of the failure of general relativity. The gravitational causal singularity is the extreme density and as a result, so intense gravity in a point of space-time where the spacetime breaks down. These causal singularities are appeared in metric solutions of the Einstein‘s gravitational field equations and they can not omitted by usual coordinate transformations. Penrose-Hawking singularity theorems show by holding the Einstein‘s metric equations under some circumstances the existence of space time singularities is unavoidable [2–4]. In order to avoid the central singularity of black holes, Penrose suggested his cosmic supervision hypothesis which says: the singularity of black hole is always hidden behind its event horizon [5]. Nevertheless, many agrees the singularity is generated by classical gravity theories, while they are neither physical nor exist in universe [6]. Sakharov [7]

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and Gliner [8] firstly showed by considering the effects of quantum, the spacetime singularity is avoidable. Bardeen [9] inspired by Sakharov‘s idea, he proposed the first singularity free solution of black holes which are called as regular black holes now. He suggested a static spherically symmetric solution without considering a known physical source. Later, numerous different kinds of regular black holes were suggested [10–14]. Among them, Ayon-Beato and Garcia (ABG) [15] considered a nonlinear electromagnetic field as a physical source to produce regular black holes. In this way, they obtained ABG regular black holes by solving the Einstein‘s metric equations which are coupled with suitable nonlinear electromagnetic fields. By following this method, other authors also confirmed ABG regular black holes [16–18]. Cai and Miao [19] achieved a kind of generalized ABG related black hole solutions which are dependent on five parameters named mass, charge and three parameters related to nonlinear electrodynamic fields. This kind of black hole returns to regular black hole under special conditions. Also, ABG black hole [15] and its other generalization [20] is obtained under some assumptions. In [19] a new family of ABG black holes have been focused which have three parameters named mass, charge and dimensionless parameter gamma. Cai and Miao [19] studied quasinormal modes and shadows radius for this new family of ABG black hole and also analyzed the effects of charge and gamma on event horizon radius and Hawking temperature. Hawking by considering quantum effects, showed black holes radiate like black bodies with particular temperature [21], related to surface gravity of black holes horizon and Bekenstein attributed entropy to black holes which is related to area of surface of the black holes horizon as $S = A/4$ [22]. These two discoveries lead us to investigate the thermodynamic behavior of black holes. Black hole thermodynamics is the consequence of relation between general relativity and quantum field theory which guide us to the unknown quantum gravity. By considering black holes as thermodynamic systems, Bardeen, Carter and Hawking [23] rewrote the four laws of thermodynamic for black holes. In this way, Davies studied the phase transition of Kerr black hole in [24]. In study thermodynamics of the black holes in usual way we need pressure thermodynamic coordinate which is bring from cosmological constant in extended phase space with negative value. In fact this is originate from CFT/AdS correspondence where we investigate to study thermodynamics of the black holes. In this way the Hawking and Page discovered a first order phase transition for black holes in Anti-de Sitter (AdS) spacetime [25]. Other types of phase transitions have been followed in other works [26–31]. Since the cosmological constant has been suggested as thermodynamic pressure [32–35], the attentions have been attracted to black hole thermodynamics in extended phase space [36–42]. Layout of this work is as follows.

In section 2 we define metric of generalized ABG black holes briefly. In section 3 we investigate thermodynamics perspective of the model. In section 4 we study possibility of the black hole phase transition. Section 5 is dedicated to summary and conclusion.

2 Generalized ABG black hole

Let us we start with the following nonlinear Einstein Maxwell action functional [15]

$$S = \int d^4x\sqrt{-g}\Big[\frac{R}{16\pi} - \frac{L(P)}{4\pi}\Big] \tag{2.1}$$

in which, $R = g_{\mu\nu}R^{\mu\nu}$ is Ricci scalar and $g = |detg_{\mu\nu}|$ is absolute value of determinant of metric tensor field. Nonlinear electromagnetic field lagrangian density $L(P) = 2PH_P - H(P)$ is coupled as minimally with the gravity where $P \equiv \frac{1}{4}P_{\mu\nu}P^{\mu\nu}$ is a gauge invariant scalar. $P_{\mu\nu} \equiv \frac{F_{\mu\nu}}{H_P}$ is nonlinear antisymmetric tensor versus the electromagnetic tensor field $F_{\mu\nu} \equiv \partial_\mu A_\nu - \partial_\nu A_\mu$, where A_μ is electromagnetic potential. $H(P)$ is a structure function of nonlinear electrodynamic field and $H_p = \frac{dH(P)}{dP}$ [15]. By looking at the ref. [19], one can infer that the above model has a spherically symmetric static black hole metric field as,

$$ds^2 = -f(r)dt^2 + f(r)^{-1}dr^2 + r^2(d\theta^2 + \sin\theta^2 d\varphi^2). \tag{2.2}$$

in which

$$f(r) = 1 - \frac{2mr^{\frac{\alpha\gamma}{2}-1}}{(q^\gamma + r^\gamma)^{\alpha/2}} + \frac{q^2 r^{\frac{\beta\gamma}{2}-2}}{(q^\gamma + r^\gamma)^{\beta/2}} \tag{2.3}$$

is called generalized ABG black hole metric potential with mass parameter m and magnetic charge q and three different dimensionless parameters α, β and γ associated to nonlinear electrodynamic fields. By assuming $\alpha\beta \geqslant 6$, $\beta\gamma \geqslant 8$ and $\gamma > 0$ the solution (2.3) reduces to regular black hole solutions [9]. For particular choices $\alpha = 3$, $\beta = 4$ and $\gamma = 2$ the generalized ABG metric field (2.3) returns to original ABG black hole solution [15] and it goes to other generalized ABG black hole solutions [20] by setting $\gamma = 2$. In this work we follow a particular choice of generalized ABG black hole metric given by ref. [19], which is introduced by considering the conditions $\alpha = \frac{6}{\gamma}$ and $\beta = \frac{8}{\gamma}$ such that

$$f(r) = 1 - \frac{2mr^2}{(r^\gamma + q^\gamma)^{3/\gamma}} + \frac{q^2 r^2}{(r^\gamma + q^\gamma)^{4/\gamma}}. \tag{2.4}$$

It is easy to check that the above metric solution reduces to the well known Reissner-Nordström form of the black holes for limits $\gamma \to \infty$. In the subsequent section we study thermodynamics of the above mentioned black hole.

3 Thermodynamics of generalized ABG black hole

According to AdS/CFT correspondence it is useful to consider a negative cosmological parameter which behaves as pressure of AdS vacuum space affecting on the existent black hole in extended phase space. In order to participate a good form of cosmological parameter in the metric potential (2.4) we rewrite it as follows.

$$f(r) = 1 - \frac{2M(r)}{r} - \frac{1}{3}\Lambda(r)r^2 \tag{3.1}$$

which is similar to a modified Schwarzschild-AdS black hole solution with the mass function

$$M(r) = \frac{mr^3}{(r^\gamma + q^\gamma)^{3/\gamma}} \tag{3.2}$$

and the variable cosmological parameter

$$-8\pi P(r) = \Lambda(r) = \frac{-3q^2}{(r^\gamma + q^\gamma)^{4/\gamma}} \tag{3.3}$$

respectively in which we defined $P(r)$ to be variable pressure of the AdS space. By solving the event horizon equation $f(r_+) = 0$ we determine radius of the black hole exterior horizon $r_+ = r_+(q, m, \gamma)$ versus the black hole parameters such that

$$1 - \frac{2mr_+^2}{(r_+^\gamma + q^\gamma)^{3/\gamma}} + \frac{q^2 r_+^2}{(r_+^\gamma + q^\gamma)^{4/\gamma}} = 0. \tag{3.4}$$

Although this equation of horizons has not simple analytic form of roots for r_+ but there is not a problem and we can continue our studies about thermodynamics of this black hole. By substituting (3.2) and (3.3) into the horizon equation (3.4) we obtain the black hole ADM mass function or enthalpy as follow.

$$M(r_+) = \frac{r_+}{2} + \frac{4\pi}{3} r_+^3 P(r_+). \tag{3.5}$$

In the extended phase space the black hole mass plays the role of enthalpy of a thermodynamic system $M = H = U + PV$, which in comparison to (3.5), the inertial energy U and thermodynamic volume V correspond with the following forms respectively.

$$U(r_+) = \frac{r_+}{2}, \quad V(r_+) = \frac{4\pi}{3} r_+^3. \tag{3.6}$$

However for this black hole the thermodynamic volume is obtained as equal to the geometric volume of the black hole but these are two different quantities and have different forms in many black holes. In fact thermodynamic volume of a black hole system is conjugate quantity for the pressure in the black hole equation of state. By regarding the first law in ordinary thermodynamic systems such that $TdS = dU + PdV$ and utilize the thermodynamic quantities attained earlier (eq. (3.6)), we define the Bekenstein entropy for this modified ABG black hole as follows.

$$S(r_+) = \int \left[\frac{1}{2} + 4\pi r_+^2 P(r_+)\right] \frac{dr_+}{T(r_+)} \tag{3.7}$$

in which $T(r_+)$ is the black hole Hawking temperature given by surface gravity of the exterior event horizon r_+ [19] as follows.

$$T(r_+) = \frac{f'(r_+)}{4\pi} = \frac{r_+^\gamma \left[1 - q^2 r_+^2 (r_+^\gamma + q^\gamma)^{-4/\gamma}\right] - 2q^\gamma}{4\pi r_+ (r_+^\gamma + q^\gamma)} \tag{3.8}$$

where we eliminated m by substituting the horizon equation (3.4). In the next section we calculate the modified ABG black hole equation of state and investigate possibility of thermodynamic phase transitions.

4 Equation of state and phase transitions

By regarding positivity condition on the thermodynamic specific volume of the black hole fluid we must be choose

$$\gamma = 2n, \quad n = \pm 1, \pm 2, \pm 3, \cdots, \quad r_+ = -xq, \quad r_+ > 0, \quad x > 0, \quad q < 0 \tag{4.1}$$

in the Hawking temperature (3.8) such that

$$t = pv + \frac{2 + x^{2n}}{4\pi x(1 + x^{2n})} \tag{4.2}$$

where we defined dimensionless the temperature t, the pressure p and the specific volume v respectively as follows.

$$t = qT, \quad p = q^2 P = \frac{3}{8\pi(1 + x^{2n})}, \quad v = \frac{2}{3}\left(\frac{x^{1+2n}}{1 + x^{2n}}\right) \tag{4.3}$$

We should pointed that the choices $q > 0$ for which $r_+ < 0$ corresponds to analytic continuation of the horizon radius in the complex analytic algebra which we do not consider them in this paper. However these part of solutions can be physical because the Bekenstein Hawking entropy of the black hole has a relationship with square exponent of the horizon radius (the horizon surface area). One can check that the above equation of state reaches to a pressureless dust fluid at large scales

$$\lim_{x\to\infty} v \approx \frac{2x}{3}, \quad \lim_{x\to\infty} p \approx 0, \quad t \approx \frac{1}{3\pi v} \tag{4.4}$$

but for small scale black holes $x \to 0$ it is possible to appear phase transitions. To study this phenomena we must be have critical points in phase space. The equation of state in ordinary thermodynamics systems has a great importance, so does it in black hole thermodynamics. The reason is, this equation refers to the thermodynamic behavior of the thermodynamic systems, in this case, the black holes. Further more, it helps us to calculate the critical points and explore the black holes' behavior at these points. Now that, we have the equation of state, we are capable to find critical quantities through solving below equations:

$$\left.\frac{\partial t}{\partial v}\right|_p = 0, \quad \left.\frac{\partial^2 t}{\partial v^2}\right|_p = 0. \tag{4.5}$$

By using chain rule in the derivatives the above critical equations reduce to the conditions $\left.\frac{\partial t}{\partial x}\right|_p = 0$ and $\left.\frac{\partial^2 t}{\partial x^2}\right|_p = 0$ with the following parametric critical points

$$p_c = \frac{3}{8\pi}\left(\frac{x_c^{4n} + (2n+3)x_c^{2n+2} + 2}{x_c^{4n+2} + (2n+1)x_c^{2n+2}}\right) \tag{4.6}$$

$$v_c=\frac{2}{3}\left(\frac{x_x^{1+2n}}{1+x_c^{2n}}\right) \tag{4.7}$$

and

$$t_c=\frac{x_c^{4n}+(2n+3)x_c^{2n}+2n+2}{2\pi x_c(1+x_x^{2n})(x_c^{2n}+2n+1)} \tag{4.8}$$

where numeric values of the quantities $\{n,x_c\}$ are obtained by the following equation.

$$x_c^{8n}+(5+6n)x_c^{6n}+(9+18n+4n^2)x_c^{4n}+(7+18n+8n^2)x_c^{2n}+2+6n+4n^2=0. \tag{4.9}$$

Diagram of the equation (4.9) is plotted in figure 1-a and it shows that for $x_c>0$ we must be choose $n=-1,-2,-3,\cdots$. In the table 1 we collected numeric values for critical points for samples $n=-1,-2,\cdots-10$. and they use to plot other diagrams. The figure 1-b shows that by raising $|n|$ the critical specific volume of the black hole gas/fluid increases but corresponding the critical pressure and the critical temperature decrease. By substituting

n	x_c	v_c	p_c	t_c
-10	0.908505	0.528167	1.15869	0.160807
-9	0.904823	0.517669	1.13268	0.159831
-8	0.901143	0.505222	1.0955	0.158429
-7	0.89778	0.490191	1.04048	0.156359
-6	0.895459	0.471618	0.955326	0.153179
-5	0.895958	0.447979	0.815329	0.14803
-4	0.904174	0.416661	0.565345	0.139054
-3	0.936833	0.372637	0.0643245	0.121634
-2	0.589602	0.350688	4.82914	0.32111
-1	0.57735	0.288675	8.24668	0.413497

Table 1: Critical points for several different values of n.

(4.3) into the entropy (3.7) we obtain dimensionless entropy as follows.

$$s=\frac{S}{q^2}=-\int\frac{6\pi x(1+x^{2n})[1+8\pi x^2p]}{8\pi px^{2+2n}+3x^{2n}+6}dx \tag{4.10}$$

and Gibbs free energy $G=M-TS$ reads

$$g=\frac{G}{q}=-\frac{\mu x^3}{(1+x^{2n})^{\frac{3}{2n}}}-t(x)s(x),\quad \mu=\frac{m}{q}. \tag{4.11}$$

and heat capacity at constant pressure $c_p=t\left(\frac{\partial s}{\partial t}\right)_p$ is given by

$$c_p=\frac{-6\pi x^2(1+8\pi px^2)}{(1+x^{2n})(8\pi px^{2+2n}+3x^{2n}+6)}\times \tag{4.12}$$

$$\frac{(8\pi p x^{2+2n}+24\pi p x^{2+4n}+24\pi p x^{2+6n}+8\pi p x^{2+8n}+6+21x^{2n}+27x^{4n}+15x^{6n}+3x^{8n})}{(8\pi p x^{2+4n}+16\pi p n x^{2+2n}+8\pi p x^{2+2n}-3x^{4n}-6nx^{2n}-9x^{2n}-6)}$$

We plotted diagrams of the above thermodynamic variables in figures 2,3,4, and 6. All figures of 2-a,2-b,2-c and 2-d show that for pressures less than the critical one $p < p_c$ diagrams of the temperature vs the specific volume at constant pressure behave as an ideal gas and so the system can not participate in the phase transition. But for $p \geq p_c$ the black hole gas/fluid system participates in a small to large black hole phase transition. In fact stable point of the disequilibrium black hole gas/fluid is minimum point in the $t - v$ diagram at constant pressure. For small scale black holes where $0 < x < 1$ by comparing the diagrams 2-a with 2-b one can infer that the black hole takes on just two phase means that it is made from two subsystems. But by raising $|n|$ there is appeared third phase. One can compare the figures 2-b with 2-d to infer that by raising $|n|$ the third phase is appeared also in small specific volumes for big black holes $0 < x < 10$. The figures 3 show pressure -specific volume diagrams at constant temperatures. They show that for temperatures higher than the critical one $t > t_c$ the black hole thermodynamic system participates in the Hawking-Page phase transition where a black hole evaporates completely to reach the AdS vacuum. This is because to have a maximum point for $p - v$ diagrams. The figures 3-b and 3-d shows third phase for the system which behaves like the ideal gas. Diagrams of 4 shows variations of the heat capacity at constant pressure versus the specific volume. Changing the sign of these diagrams by rasing v means that the phase transition is appeared for the black hole gas/fluid system. There is two position where the phase transition appears just for $p \leq p_c$ By comparing these diagrams one can infer that possibility to bing the phase transition is more for small $0 < x < 1$ and large $0 < x < 10$ black hole just by increasing $|n|$ factor. The diagrams in figure 5 show variations of the free Gibbs energy vs the specific volume. 5-a and 5-c show that for small black holes $0 < x < 1$ when $p < p_c$ then the black hole takes on two phase just for limited scales means for particular scales in the specific volume the black hole has two different values for the Gibbs energy but for large scale black holes $0 < x < 10$ this free from the black hole scales. In other words the black hole for all scales has two different phases. One should to look diagram of 5-d where the crossing point of the $g - v$ diagram makes a swallowtail form. This means coexistence of the two different phase where the black hole is in equilibrium with them. This is appeared just for $p \leq< p_c$. All diagrams in figure 5 are plotted for $\mu = \frac{m}{q} = -10$ and for $\mu = +10$ they are repeated in the figures 6. In the latter case there is not crossing point and so coexistence state for the two different phases of the black hole gas/fluid system.

5 Conclusion

By re-defining the charge quantity of a generalized Ayon-Beato-Carcia nonsingular black hole vs the variable cosmological parameter we obtained its thermodynamic equation of state. The cosmological parameter is needed to consider pressure of AdS vacuum space effecting on thermodynamic behavior of a black hole. By studying isothermal P-v curves we infer that the system participates in the Hawking-Page phase transition where the disequilibrium evaporating ABG black hole reaches finally to a vacuum AdS space. Other diagrams such that Gibbs free energy and heat capacity satisfy possibility of phase transition and there is also a coexistence point in phase space depended to electromagnetic field coupling constant γ where the two different phases exist synchronously. For small scale black holes there are three phases while for larger than there are just two phase. As extension of this work we like to study other thermodynamic behavior of the modified ABG black hole such as heat engine and Joule-Thomson expansion and etc.

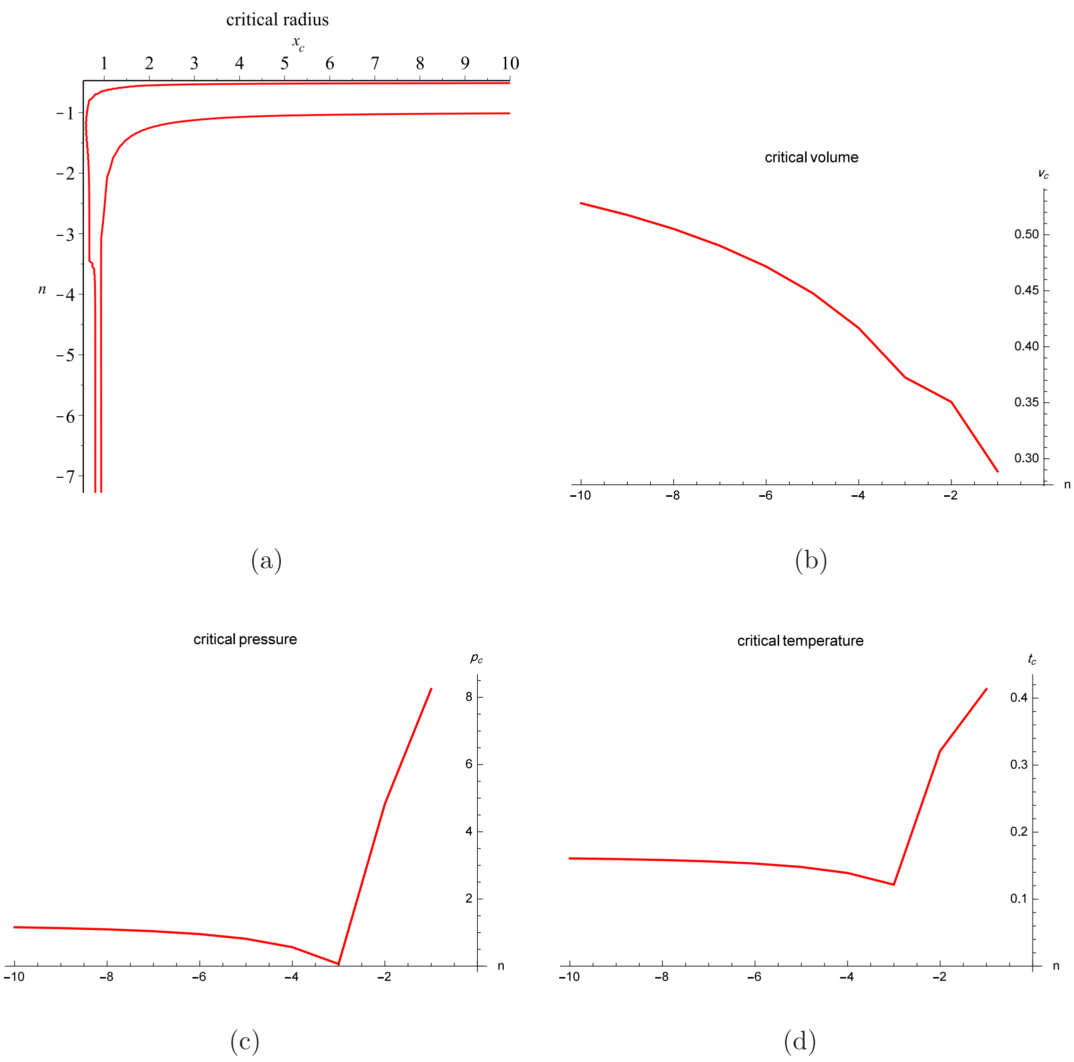

Figure 1: Numeric diagrams for x_c, v_c, p_c, t_c versus n

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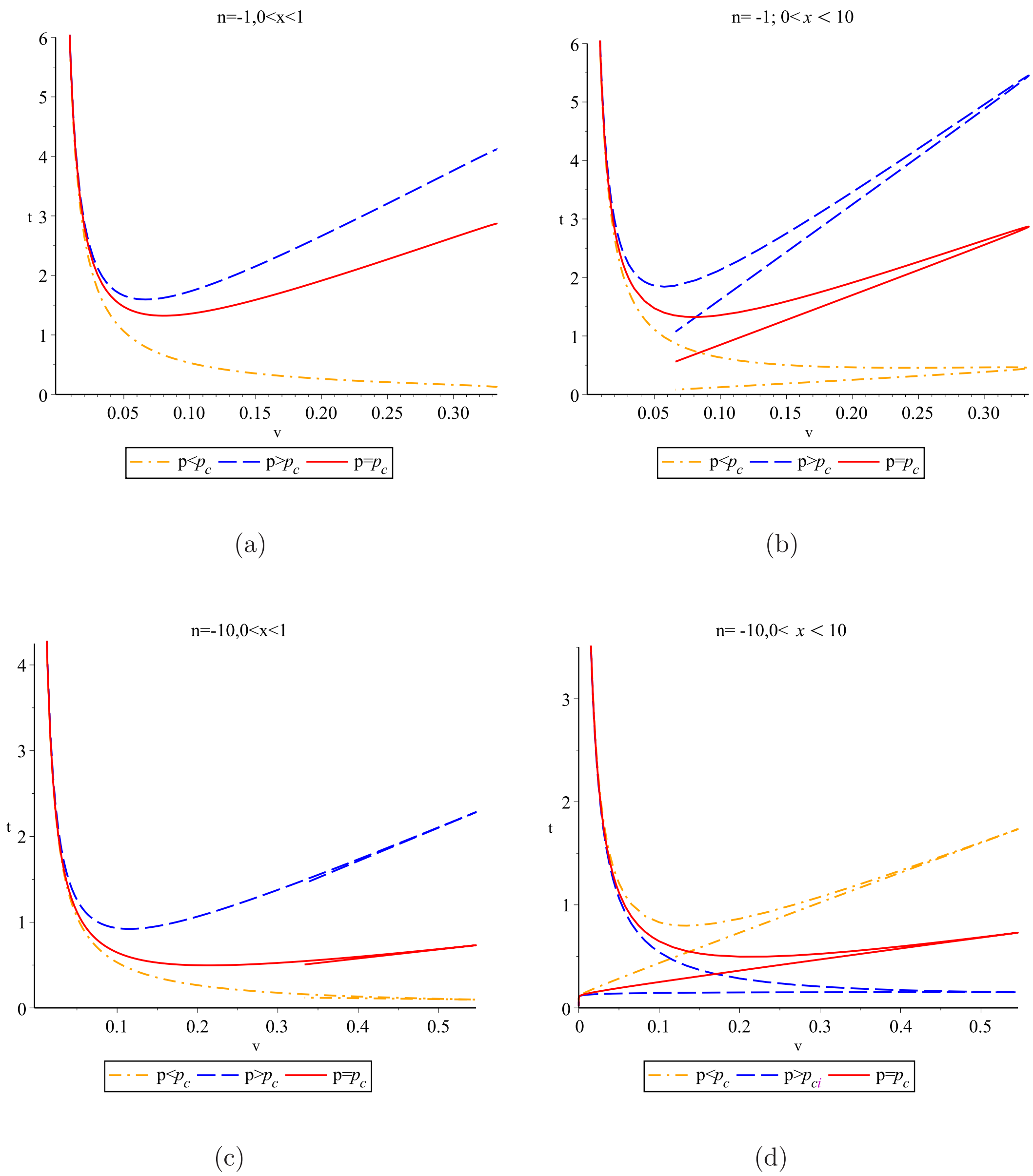

Figure 2: t-v diagrams at constant pressure

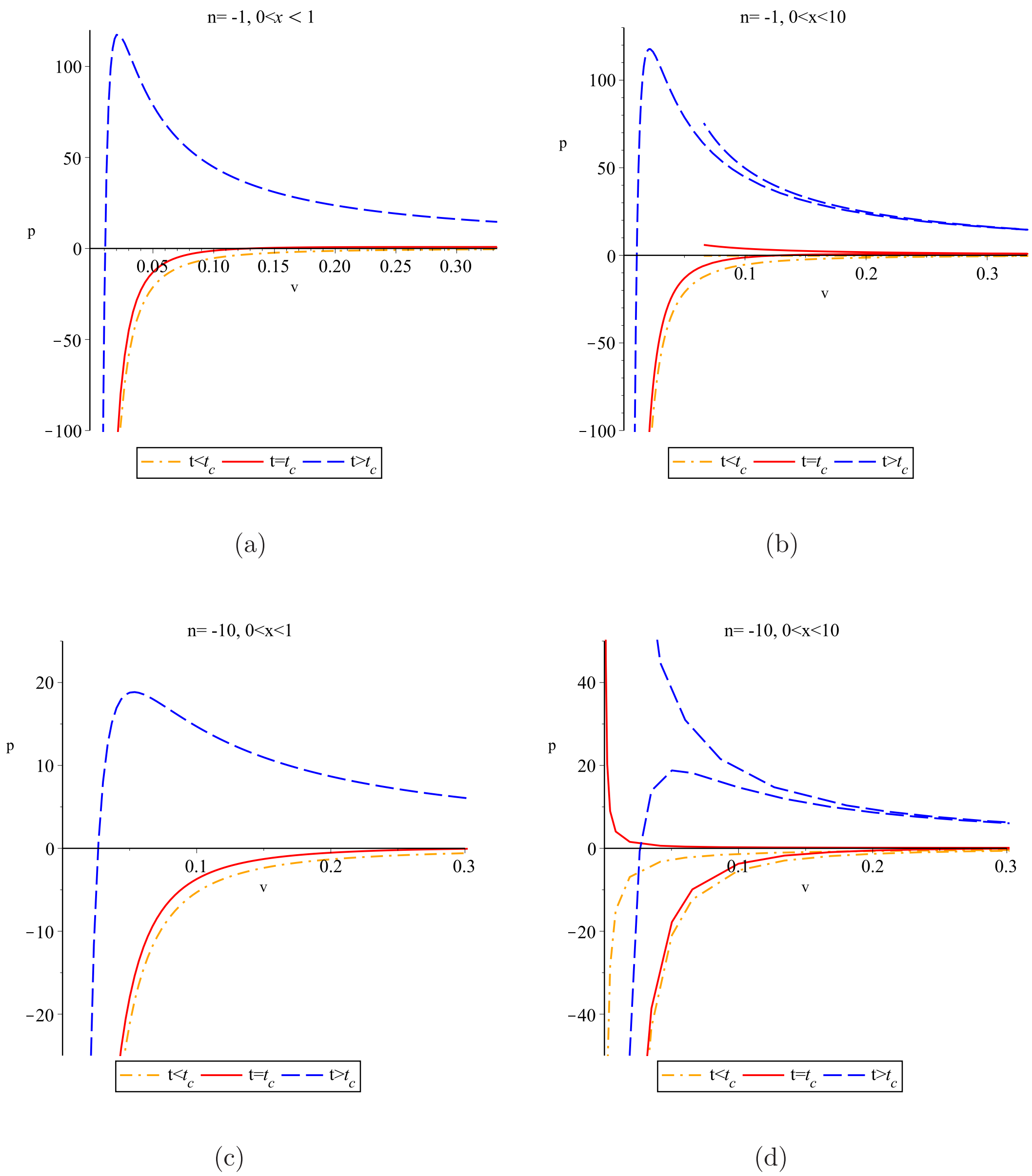

Figure 3: p-v diagrams at constant temperature

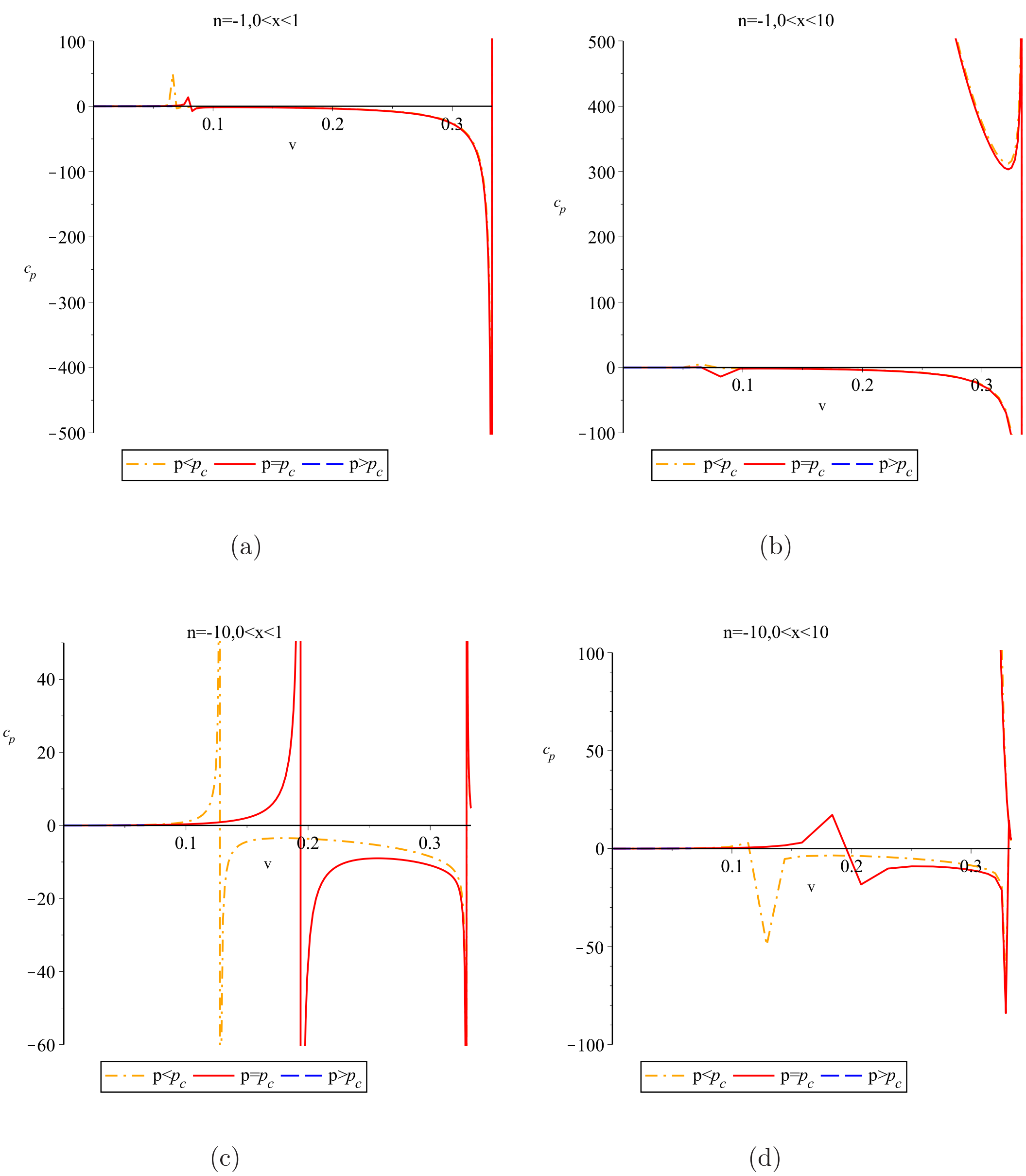

Figure 4: $c_p - v$ diagrams

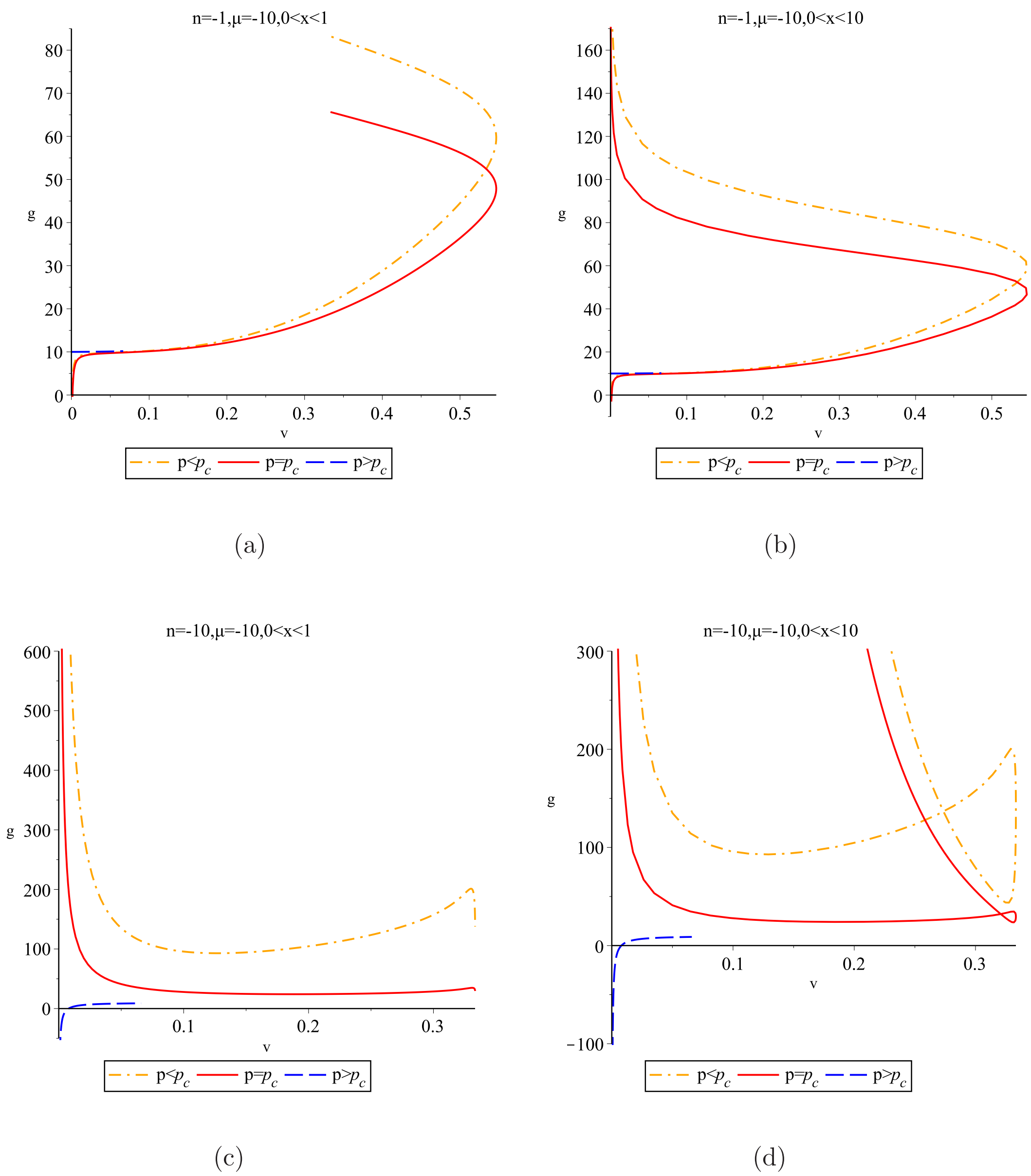

Figure 5: $g - v$ diagrams for $\mu = -10$; $(m > 0, q < 0)$

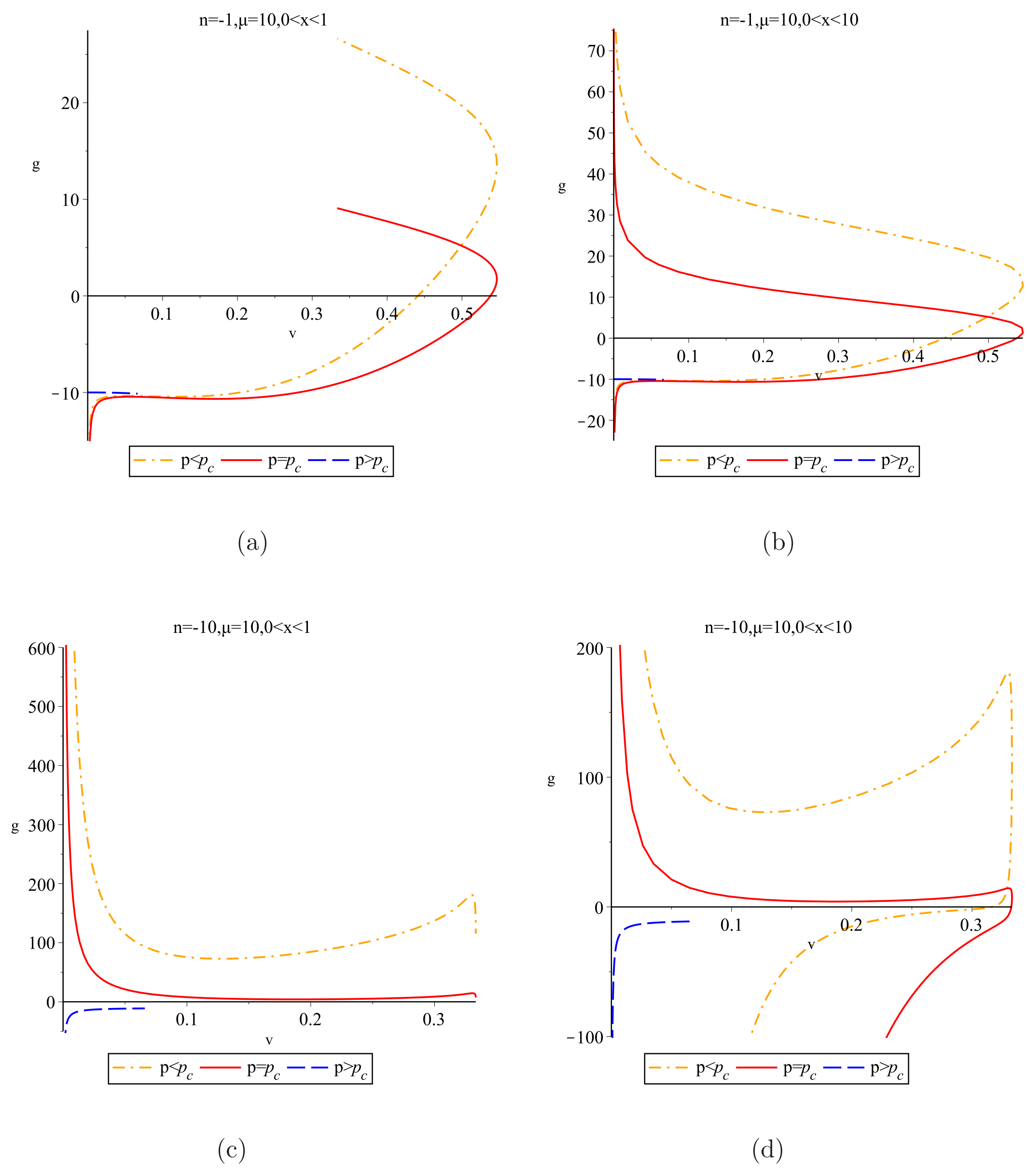

Figure 6: $g - v$ diagrams for $\mu = 10; (m < 0, q < 0)$